INSTRUCTIONS:

- Computers, calculators, books and notes are not permitted.
- An 8.5x11 inch crib sheet is allowed.
- Write your name and instructor's name on the front of the quiz.
- Show your work and clearly identify your final answer.

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1. (25 points)

   The Logistic Equation for population growth is

   $$\frac{dy}{dt} = ry - \frac{r}{L}y^2, \quad y \geq 0.$$

   (a) Find and classify the equilibrium points and their stability. Use this to draw a *phase line*, or a *direction field* for this differential equation.

   (b) Let $r = 1$ and $L = 4$. Solve the differential equation with the initial condition $y(0) = 1$.
   (You must show the steps of how you solve this differential equation, even if you know the solution).

2. (30 points)

   Salt water with a concentration of 0.5 pounds per gallon is pumped into a tank at a rate of 4 gallons per minute. Initially, the 150 gallon tank has 50 gallons of 2 pound per gallon salt water. The well stirred mixture flows out of the tank at a rate of 2 gallons per minute.

   (a) How long until this tank overflows?

   (b) Set up an initial value problem satisfied by the amount of salt in the tank.

   (c) Find the amount of salt in the tank at any time until the tank overflows.

3. (50 points)

   Solve the following differential equations:

   (a) $\frac{dy}{dt} = \frac{75}{4} - \frac{t}{8} - \frac{y}{4}, \quad y(0) = 35$

   (b) $\frac{dy}{dt} = \frac{\sin(t)}{8y+4}, \quad y(0) = -1$

   (c) $y' + \frac{y}{t} = \sqrt{1+t^2}$

   (d) $y' = 2t\frac{y}{t^2} + \frac{t^2}{y}, \quad y(1) = 2$ (You may find the substitution $v(t) = \frac{y(t)}{t^2}$ extremely helpful).

4. (25 points)

   (a) Is the set of all pairs of real numbers (x, y) such that $xy \geq 0$ a vector space?

   (b) Is the set of all solutions of the differential equation $y' + (e^t + y)t + y = 0$ a vector space?

   (c) The *Trace* of an $n \times n$ matrix is the sum of the diagonal elements, Tr $\mathbf{A} = a_{11} + a_{22} + \cdots + a_{nn}$. Is the set of all 2×2 matrices with zero trace a *subspace* of the set of all 2×2 matrices? If so, give a *basis* for this subspace. If not, give an example of why.

   (d) A matrix $\mathbf{A}$ is said to be *skew-symmetric* if $\mathbf{A}^T = -\mathbf{A}$. For example, the matrix $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ is skew-symmetric. Is the set of all 2×2 skew-symmetric matrices a vector space?

5. (15 points)

Consider the linear system of equations

$$\begin{aligned} x + hy &= 1 \\ hx + y &= -1. \end{aligned}$$

Determine all values of h so that this system has
(a) a unique solution
(b) no solutions
(c) infinitely many solutions.

6. (25 points)

Does the set of polynomials $\left\{t^3 - t + 2,\ -t^3 + t^2 - 3,\ 2t^2 + 3t - 1,\ t^3 - 6t + 1\right\}$ span $\mathbb{P}_3$, the set of all polynomials of degree less than or equal to 3? You must justify your answer.

7. (30 points)

Consider the system of equations

$$\begin{aligned} 3x_1 + 3x_2 + 3x_3 &= 0 \\ 2x_1 + 2x_2 + 5x_3 - 3x_4 &= 0 \\ x_1 + x_2 + 2x_3 - x_4 &= 0. \end{aligned}$$

(a) Solve this system of equations.

(b) What is a basis for the subspace of $\mathbb{R}^4$ spanned by the solution?

(c) Show that the basis vectors in part (b) are linearly independent.