
INSTRUCTIONS: Computers, calculators, books, notes, flying monkeys, *etc.* are not permitted. Some (possibly) useful formulae are attached. Write your name, your instructor's name, and the color of your exam sheet on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

---

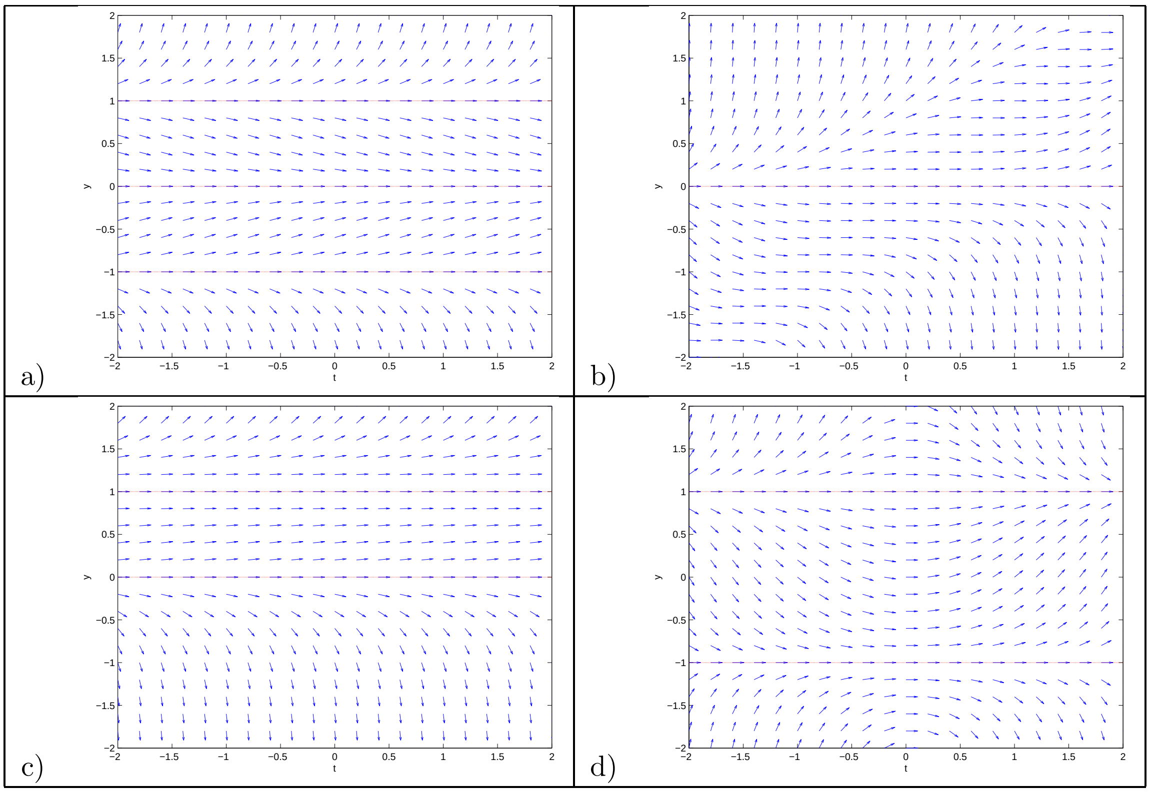
1. Solve the following IVPs using any appropriate method.
   (a) $w' = w^2 \sin(x)$, $w(0) = 1/5$.
   (b) $x' - \tan(t)x = t$, $x(0) = 0$.

2. The body of a murder victim was discovered at 11:00 A.M. The medical examiner arrived at 11:30 A.M. and found the temperature of the body was 94.6°F. The temperature of the room was 70°F. One hour later, in the same room, she took the body temperature again and found that it was 93.4°F. Assuming that Newtons law of cooling holds, *i.e.* $T' = k(T_{\text{room}} - T)$, estimate the time of death, given that usual body temperature is 98.6°F. (Note: without a calculator, you will not be able to give an actual time — give your answer in terms of calculable quantities, such as "$\sqrt{12.4}$ minutes before 11 A.M." or "sin(0.76) hours after 11:30".)

3. (a) Solve the IVP $\frac{dy}{dt} = \sin(\sqrt{y})e^{-t^2}$, $y(0) = 0$. [Hint: very easy! But remember to justify your answer.]
   (b) Suppose $\mathcal{L}$ is a linear operator, u is a solution of $\mathcal{L}[y] = f(t)$, v is a solution of $\mathcal{L}[y] = g(t)$ and w is a solution of $\mathcal{L}[y] = 0$. What is the general solution of $\mathcal{L}[y] = h(t)$ when $h(t) = f(t) + \pi g(t)$?
   (c) Does $x' = |x| - t^2$ have any solutions? If so, do the solutions cross anywhere?
   (d) Determine the long-term ($t \to \infty$) behavior of the solution to $y' = y\cos(\pi y/4)$, $y(0) = 1$. Be sure to justify your claim.

4. Consider the linear, non-homogeneous equation $z'' = \frac{2t}{t^2+4}(z' - 1)$.
   (a) Find a particular solution of the form $z(t) = At + B$, where A and B are some unknown constants (that you should determine).
   (b) Show that $z(t) \equiv 1$ is a solution of the homogeneous part of the equation.
   (c) Find another homogeneous solution, by first defining $y = z'$ and solving the corresponding equation for y.
   (d) Determine the general solution to the non-homogeneous equation, using your answers to (a)–(c). Simplify your final answer.

5. Match each of the pictures shown (a)–(d) with one of the equations below. (Note: there are more equations than pictures, so five equations will be unused.) No work need be shown for this problem.



(1)	$y' = t(y^2 - 1)$	(2)	$y' = y(t^2 - y)$	(3)	$y' = y(y-1)^2$
(4)	$y' = t(y - t)^2$	(5)	$y' = y(1 - y^2)$	(6)	$y' = y(y^2 - 1)$
(7)	$y' = y(y - t)^2$	(8)	$y' = t(y^2 - t)$	(9)	$y' = t(1 - y^2)$