INSTRUCTIONS: Computers, calculators, books, notes, frying monkeys, *etc.* are not permitted. Write your name, your instructor's name, and your recitation section number on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. True or False. State whether the following statements are (always) "TRUE" or "FALSE" (meaning not always true). You MUST write the full word TRUE or FALSE — T/F will NOT be graded. For this question only you do NOT need to show your working or reasoning.
   (a) If A, B and $A+B$ are all invertible matrices, then $(A+B)^{-1} = B^{-1} + A^{-1}$.
   (b) If $AX = XB$ and $\det(X) \neq 0$, then $A = B$.
   (c) The solution of 3 linear algebraic equations in 2 variables can represent the intersection of 3 lines in the plane.
   (d) If $A = B^{-1}CB$ (where B is some invertible matrix) then $\det(A) = \det(C)$ even though A is not necessarily equal to C.
   (e) If $\det(A) = 0$ then the system of equations $A\mathbf{x} = \mathbf{b}$ has no solutions.

2. For which value(s), if any, of λ does the system of equations

$$\begin{aligned} \lambda x + z &= 3 \\ 2x - 3y &= 4 \\ x - y + z &= 2 \end{aligned}$$

   (a) have a unique solution?
   (b) have no solutions?
   (c) have infinitely many solutions?

3. Let

$$A = \begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix}$$

   (a) Find A^{-1} or explain why it does not exist.
   (b) Use your answer to (a) to find the solution(s) of $A\mathbf{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ or explain why no solutions exist.
   (c) Are there any vectors $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ has no solution? If so, what are the conditions on $\mathbf{b}$; if not, why not?

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4. Consider the equation $x'' = 2x' + 3x$.

   (a) Find the general solution.

   (b) Set up (but do not solve) the system of equations needed to find the particular solution that satisfies the initial condition $x(0) = \alpha$, $x'(0) = \beta$.

   (c) Are there any initial conditions that your solution cannot satisfy? Explain!

5. (a) For what value(s) of α, if any, are the vectors

$$\begin{bmatrix} \alpha \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ \alpha \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ \alpha \end{bmatrix}$$

   in $\mathbf{R}^3$ linearly **dependent**?

   (b) Find all solutions of the system of equations

$$\begin{aligned} x + y + z &= 0 \\ y - z &= 0 \end{aligned}$$

   (c) Let W be the solution space (*i.e.* the vector space of solutions) of the equations in (b). Using your answer to (b), find a basis and the dimension of W. [Hint: think of your answer to (b) as the span of a basis set.]

6. (EXTRA CREDIT — 4 points, **no partial** credit) Prove that, if A is invertible, then the inverse is unique. That is, prove that any two matrices that are the inverse of A must be equal.