1 a) TRUE b) FALSE c) TRUE d) TRUE e) FALSE

2 a) $\det(A-\lambda I) = \lambda^2 - 2\lambda - 35 = (\lambda-7)(\lambda+5) = 0$

$\Rightarrow \lambda = -5, 7$

$\lambda = -5: \left(\begin{array}{cc|c} 6 & 12 & 0 \\ 3 & 6 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow v_1 + 2v_2 = 0$

$\Rightarrow \vec{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\lambda = 7: \left(\begin{array}{cc|c} -6 & 12 & 0 \\ 3 & -6 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} -1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right) \quad \vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

So eigenvalues/vectors are $\boxed{\left\{-5, \begin{pmatrix} 2 \\ -1 \end{pmatrix}\right\} \ \& \ \left\{7, \begin{pmatrix} 2 \\ 1 \end{pmatrix}\right\}}$

b) $\vec{x}(t) = c_1\bar{v}_1 e^{\lambda_1 t} + c_2 \bar{v}_2 e^{\lambda_2 t}$

$\Rightarrow \boxed{\vec{x}(t) = c_1\begin{pmatrix} 2 \\ -1 \end{pmatrix} e^{-5t} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{7t}} = \begin{pmatrix} 2c_1 e^{-5t} + 2c_2 e^{7t} \\ -c_1 e^{-5t} + c_2 e^{7t} \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

c) $\vec{x}' = \begin{pmatrix} -10c_1 e^{-5t} + 14c_2 e^{7t} \\ 5c_1 e^{-5t} + 7c_2 e^{7t} \end{pmatrix}$

$A\vec{x} = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 12y \\ 3x + y \end{pmatrix} = \begin{bmatrix} 2c_1 e^{-5t} + 2c_2 e^{7t} + 12(-c_1)e^{-5t} + 12c_2 e^{7t} \\ 6c_1 e^{-5t} + 6c_2 e^{7t} - c_1 e^{-5t} + c_2 e^{7t} \end{bmatrix}$

$= \begin{bmatrix} -10c_1 e^{-5t} + 14c_2 e^{7t} \\ 5c_1 e^{-5t} + 7c_2 e^{7t} \end{bmatrix} = \vec{x}'$ ✓

d) In general $c_2 \neq 0$ so solution is dominated by $c_2\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{7t}$ as $t \to \infty$

Since $e^{7t} \to \infty$, $\boxed{|\vec{x}(t)| \to \infty \text{ as } t \to \infty}$

3 a) $y'' - 8y' + 15y = 0$ Try $y = e^{rt}$ $\Rightarrow r^2 - 8r + 15 = 0$

$\Rightarrow (r-5)(r-3) = 0$

$\Rightarrow r = 3, 5$

$\Rightarrow \boxed{y_h(t) = C_1 e^{3t} + C_2 e^{5t}}$

b) i) Ae^{5t} is part of y_h $\Rightarrow \boxed{y_p = Ate^{5t}}$ $(A = \text{const})$

ii) $\boxed{y_p = (At+B)(C\cos(3t) + D\sin(3t))}$ $(A, B, C, D = \text{const.})$

iii) $\boxed{y_p = At^2 + Bt + C}$ $(A, B, C = \text{const.})$

iv) $\boxed{y_p = A\cos(3t) + B\sin(3t) + Ce^{-3t}}$ 〃

(e^{-3t} is <u>not</u> part of y_h)

v) $\boxed{y_p = At + B + Ce^{8t}}$ 〃

(e^{8t} is not part of y_h)

4 a) $x'' + 2x' + 10x = 0$ Try $x = e^{rt}$ $\Rightarrow r^2 + 2r + 10 = 0$

$\Rightarrow r = \frac{-2 \pm \sqrt{4-40}}{2} = -1 \pm \sqrt{1-10} = -1 \pm \sqrt{-9} = -1 \pm 3i$

$\Rightarrow \boxed{x_h(t) = e^{-t}(c_1 \cos(3t) + c_2 \sin(3t))}$

b) Get particular sol$^{\underline{n}}$ by Undetermined Coefficients

$x_p(t) = Ae^{-t}$

$\Rightarrow x_p' = -Ae^{-t}$ & $x_p'' = Ae^{-t}$

$\Rightarrow x_p'' + 2x_p' + 10x_p = e^{-t}(A - 2A + 10A) = 9Ae^{-t} = 3e^{-t}$ $\Rightarrow \underline{A = 1/3}$

$\Rightarrow x_p = \frac{1}{3}e^{-t}$ $\Rightarrow \boxed{x(t) = e^{-t}(c_1 \cos(3t) + c_2 \sin(3t) + 1/3)} = x_h + x_p$

c) $x(0) = c_1 \cos(0) + c_2 \sin(0) + 1/3 = c_1 + 1/3 = 1/3$ $\Rightarrow c_1 = 0$

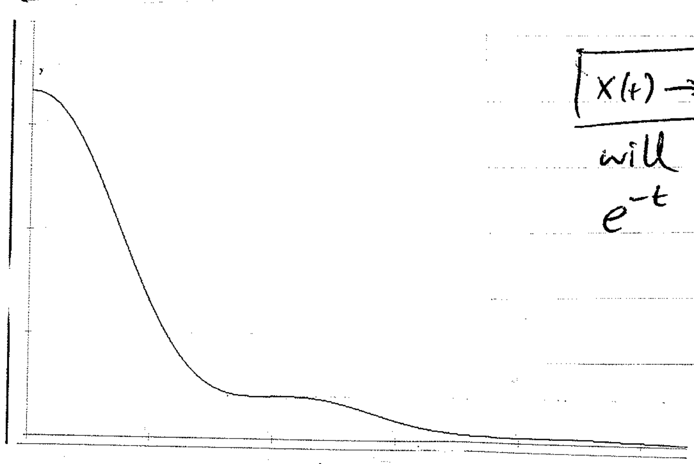
$\Rightarrow x = e^{-t}(c_2 \sin(3t) + 1/3)$

$\Rightarrow x' = -e^{-t}(c_2 \sin(3t) + 1/3) + e^{-t}(3c_2 \cos(3t))$

$\Rightarrow x'(0) = \frac{-1}{3} + 3c_2 = 0$ $\Rightarrow c_2 = 1/9$

$\Rightarrow \boxed{x(t) = \frac{1}{3}e^{-t}(\frac{1}{3}\sin(3t) + 1)}$

d)



$\boxed{x(t) \to 0 \text{ as } t \to \infty}$. All solutions will do the same b/c of the e^{-t} term (on everything)

5 a) $y = t \Rightarrow y' = 1 \Rightarrow y'' = 0$

$\Rightarrow y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 0 - \frac{2}{t} + \frac{2t}{t^2} = 0$ ✓

$y = t^2 \Rightarrow y' = 2t \Rightarrow y'' = 2$

$\Rightarrow y'' - \frac{2}{t}2t + \frac{2}{t^2}t^2 = 2 - 4 + 2 = 0$ ✓

b) Get particular sol$^{\underline{n}}$ using Variation of Parameters

$C_1' = \frac{-y_2 f}{|W|} \qquad C_2' = \frac{y_1 f}{|W|} \qquad |W| = \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} = 2t^2 - t^2 = t^2$

$C_1' = \frac{-t^2(6t^2-1)}{t^2} = 1 - 6t^2 \Rightarrow C_1 = t - 2t^3$

$C_2' = \frac{t(6t^2-1)}{t^2} = 6t - \frac{1}{t} \qquad C_2 = 3t^2 - \ln t$

$\Rightarrow y_p(t) = C_1 y_1 + C_2 y_2 = t(t-2t^3) + t^2(3t^2 - \ln t)$

$= t^2 - 2t^4 + 3t^4 - t^2\ln t = t^2 + t^4 - t^2 \ln t$

$\boxed{y(t) = C_1 t + C_2 t^2 + t^4 - t^2 \ln t}$ (absorbing t^2 term into $C_2 t^2$)

c) $y(1) = C_1 + C_2 + 1 = -1 \qquad y' = C_1 + 2C_2 t + 4t^3 - 2t\ln t - t$

$y'(1) = C_1 + 2C_2 + 4 - 1 = 3$

$\Rightarrow C_1 + C_2 = -2 \qquad \Rightarrow C_2 = 2 \Rightarrow C_1 = -4$

$C_1 + 2C_2 = 0$

$\Rightarrow \boxed{y(t) = 2t^2 - 4t + t^4 - t^2 \ln t}$