Applied Math 2360 Exam #1 Spring 2007

Instructions: This exam is closed note, closed book, closed calculator. Point totals for each question are in brackets. Show your work and BOX your answers. When in doubt about how to answer a question, keep in mind that your goal is to (as clearly as possible) communicate to us that you understand the material.

An h subscript denotes *homogeneous solution.* A p subscript denotes *particular solution.*

1. [25] TRUE/FALSE (Use the full word, "TRUE" or "FALSE". "T" or "F" will not be graded)
   (a) The differential equation $\frac{dw}{dx} = \frac{x+xw}{w+xw}$ is separable.
   (b) $z' - \sec(t)z = tz$ is a nonlinear differential equation
   (c) There exists a unique solution to the initial value problem $\{\frac{dx}{dw} = x^2; x(0) = 1\}$ in some subset of the wx-plane containing $(w,x) = (0,1)$.
   (d) The method of Variation of Parameters helps us to find an appropriate change of variable to change $y' + p(t)y = q(t)y^{\alpha}$ into a linear DE (note that this is a Bernoulli DE).
   (e) Using the Euler-Lagrange 2-step method, the general solution to $\frac{dw}{dy} = w - y$ is $w(y) = w_h(y) + Cw_p(y)$ for unknown constant C.

2. [30] Solve the following differential equations using any method at your disposal.
   (a) $\frac{dx}{dy} = x^2\cos(y); x(0) = 2$
   (b) $\frac{dy}{dx} = e^{x^{42}}(y-3)\tan(y); y(1) = 3$ [HINT: VERY EASY! Remember to justify your answer.]
   (c) $t\frac{dx}{dt} = 4t - 3x; x(1) = 2$

3. [15] Consider the differential equation

$$p' = \alpha p - \frac{\alpha}{\kappa}p^2; \, p(0) = p_0$$

   where α, κ, and p_0 are constants all greater than zero.
   (a) Draw the direction field, identifying any equilibrium solutions and their stability.
   (b) Describe the qualitative long term behavior for all initial conditions greater than zero, i.e., $p_0 > 0$.

4. [15] CSI: Boulder

   Professor Bortz left his 70° coffee in his 70° office to go teach at 10:00AM (it had been sitting there a while). Upon returning to his office (ECOT 234) at 11:00AM for office hours, he discovered that someone had opened the window, but nothing was missing! (and now the office was a chilly 40° while his coffee was 55°).

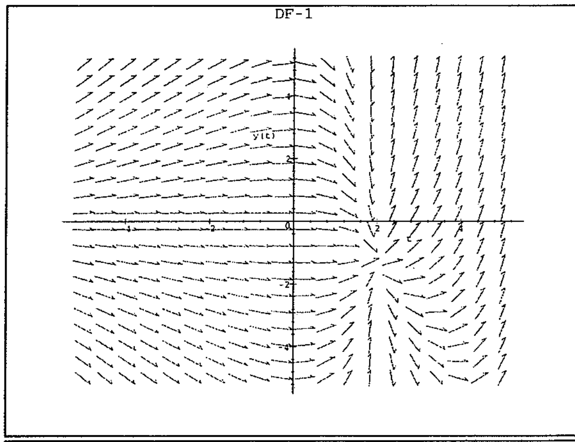
   When was the office window opened? You may assume that the office instantly became 40° when the window was opened, a good approximation for the cooling/heating coefficient (k) for coffee is ln(2) (in units of 1/hour), and that Newton's law of cooling applies here.

5. [15] Consider the following 6 linear ODE's.
ODE-I: $y' = (y/t)^2$
ODE-II: $5y' = y^3 - 3y^2 - 10y + 24$
ODE-III: $y' - 3y = 2$
ODE-IV: $10(t-2)y' = 3ty + e^t$
ODE-V: $y' + 3y = 2$
ODE-VI: $5y' = -y^3 + 3y^2 + 10y - 24$

(a) Specify the domain in t for which a unique solution exists.

(b) Match the following differential equations with the given direction fields. It is not necessary to justify your answers for 5a or 5b.

DF-1



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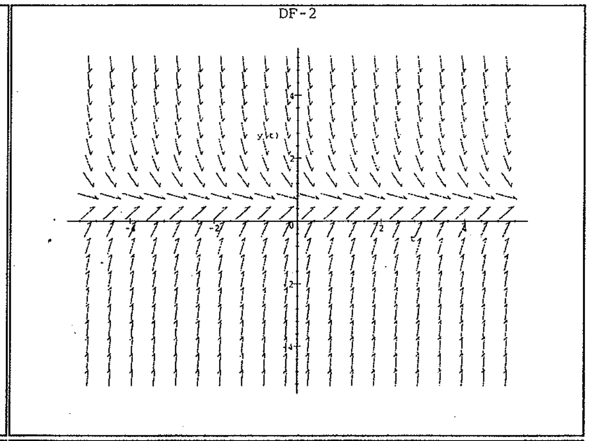

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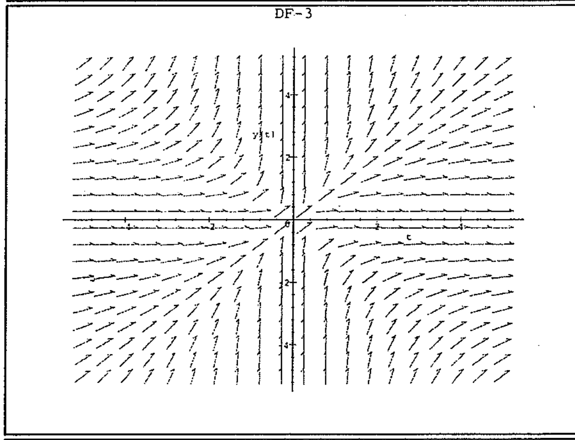

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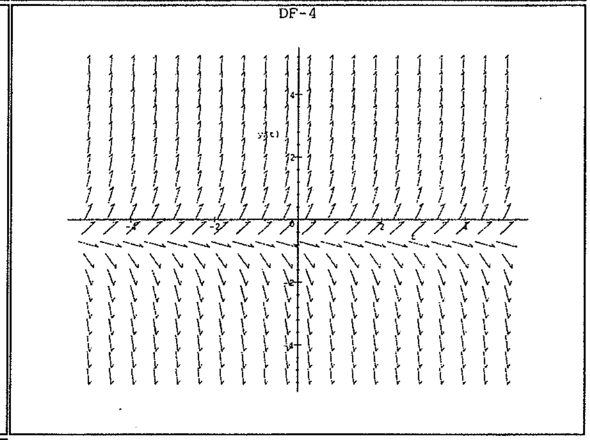

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