2360 Spring 2007
Midterm #1 Solutions

1. a) TRUE — 25 points total, 5 points per part
   b) FALSE
   c) TRUE
   d) FALSE
   e) FALSE

2. a) $\frac{dx}{dy} = x^2 \cos y$; $x(0) = 2$

8 - Soln
2 - IC

- $\int x^{-2} dx = \int \cos y \, dy$

$$-x^{-1} = \sin y + C$$

$$x(y) = \frac{-1}{\sin y + c}$$

- $x(0) = 2 = \frac{-1}{0 + c}$

$$c = -\tfrac{1}{2}$$

$$\boxed{x(y) = \frac{-1}{\sin y - \frac{1}{2}}}$$

b) $\frac{dy}{dx} = e^{x^{42}} (y-3) \tan y$; $y(1) = 3$

6 - Soln
4 - justify w/ Picard

$f(x,y) = e^{x^{42}} (y-3) \tan y$

f is continuous for all x and ~~for~~ $y = 3$
f_y is continuous for all x and for $y = 3$
$\Rightarrow$ Therefore, by Picard, we know that a solution exists and it is unique

Solution : $\boxed{y(x) = 3}$

c) $t\frac{dx}{dt} = 4t - 3x$; $x(1) = 2$

$\frac{dx}{dt} = 4 - \frac{3}{t}x$ (Solve using E-L 2-step or I.F.)

i) E-L 2-step

homog: $\frac{dx}{dt} = -\frac{3}{t}x$

$$\int \frac{1}{x}dx = \int -\frac{3}{t}dt$$
$$\ln|x| = -3\ln|t| + c$$
$$x_h(t) = Ct^{-3}$$

nonhomog: $x_p(t) = vt^{-3}$ (Var. of Par.)

$$x_p'(t) = v't^{-3} - 3v't^{-4}$$

$$\rightarrow v't^{-3} - 3vt^{-4} = 4 - \frac{3}{t}(vt^{-3})$$

$$v' = 4t^3$$
$$v = t^4$$

general:

$$x(t) = Ct^{-3} + t$$

IC: $x(1) = 2 = C + 1 \Rightarrow C = 1$

$$\boxed{x(t) = t^{-3} + t}$$

ii) I.F.

$$\mu(t) = e^{\int \frac{3}{t}dt}$$
$$= e^{3\ln|t|}$$
$$= t^3$$

$$\rightarrow \mu \frac{dx}{dt} + \mu \frac{3x}{t} = 4\mu$$

$$t^3 x' + 3t^2 x = 4t^3$$

$$\frac{d}{dt}(t^3 x) = 4t^3$$

$$\int d(t^3 x) = \int 4t^3 dt$$

$$t^3 x = t^4 + C$$

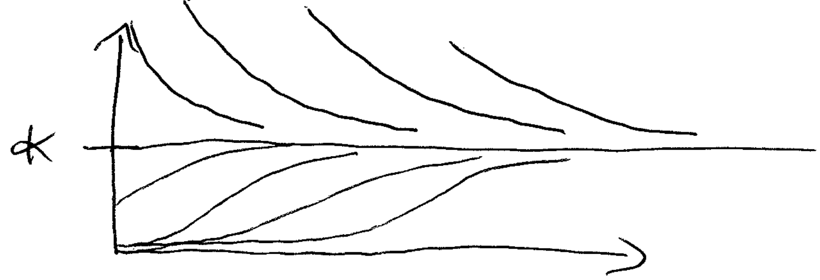
$$x(t) = t + \frac{c}{t^3}$$

IC: $x(1) = 2 = 1 + c \Rightarrow c = 1$

$$\boxed{x(t) = t + t^{-3}}$$

3. $p' = \alpha p - \alpha p^2/k \quad ; p(0) = p_0$
$= \alpha p(1 - p/k)$

5-graph a)

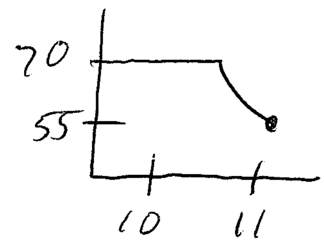


5-eqbm + stability

eqbm solns	Stability
k	stable
0	unstable

5-interp b) For all positive p_0, the solutions will asymptotically approach k as $t \to \infty$.

4.



4pts Newton's Law Eqn & Concept

$$T' = k(A - T)$$

$$T' = \ln 2(40 - T) = -\ln 2\, T + 40 \ln 2$$

7pts for solving eqn correctly

Sep. of Var., I.F., and E-L 2 step all work to obtain

$$T(t) = Ce^{-\ln 2\, t} + A$$

Apply Known condition at 11:00 AM

4 pts for applying condition and identifying when the window was opened

$$T(1) = Ce^{-\ln 2} + A = 55$$

$$Ce^{-\ln 2} + 40 = 55$$

$$C\left(\tfrac{1}{2}\right) = 15$$

$$C = 30$$

General Solution:

$$T(t) = 30e^{-\ln 2\, t} + 40$$

When was $T(t) = 70$?

$$\boxed{\text{At } t = 0 \ (10{:}00\text{AM})}$$

(It seems that it was Professor Bortz who accidentally left the window open.)

5 DF-1

ODE-IV

Solutions exist for $t \neq 2$

3 points per part

1 for the correct ~~t~~-domain

2 for correct identification,

DF-2

ODE-V

Solutions exist for all t

DF-3

ODE-I

Solutions exist for $t \neq 0$

DF-4

ODE-III

Solutions exist for all t

DF-5

ODE-II

Solutions exist for all t