

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. A one-page crib sheet is allowed.

1. (a) (15 points) Find the general solution and any singular solutions of the differential equation

$$\frac{dy}{dt} = 2\sqrt{y}$$

Are the singular solutions included in the general solution? Explain.

- (b) (5 points) Determine the values of y_0 for which the initial value problem

$$\frac{dy}{dt} = 2\sqrt{y}, \quad y(0) = y_0$$

has (i) no real solutions and (ii) a unique real solution.

2. (a) (5 points) Find the constants A and B such that

$$y_p = A \sin t + B \cos t$$

is a particular solution of the differential equation

$$\frac{dy}{dt} + y = 2 \sin t$$

- (b) (15 points) Find the general solution of the equation.

3. (21 points, 7 each) Consider the operator L defined as $L[y] = \frac{dy}{dt} + y^2$.

(a) Formally prove that L is *not* a linear operator.

(b) Make the substitution $y = 1/t + 1/u$ in the equation $L[y] = 0$ and show that $u(t)$ satisfies an equation of the form $M[u] = f(t)$. Define the operator M and the function $f(t)$.

(c) Formally prove that M is a linear operator.

4. (21 points, 7 each) A falling hailstone with density $\delta = 1$ starts from rest with negligible radius $r = 0$. Thereafter its radius is $r = kt$ (k is constant) as it grows by accretion during its fall. The initial value problem (IVP) describing the velocity $v = v(t)$ of the hailstone is

$$\frac{d}{dt}[m(t)v(t)] = m(t)g, \quad v(0) = 0$$

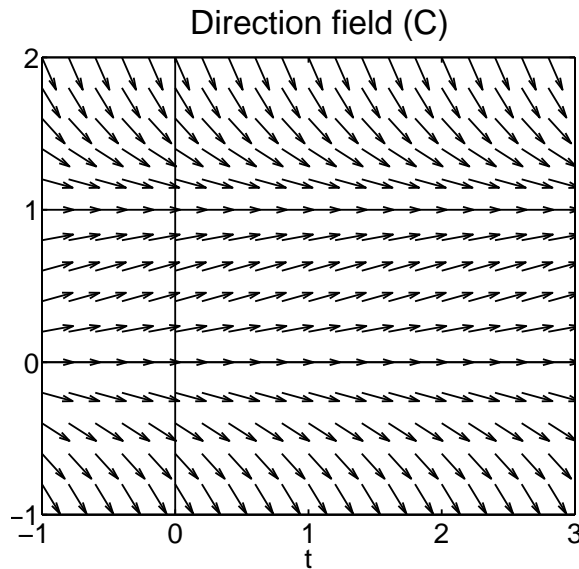
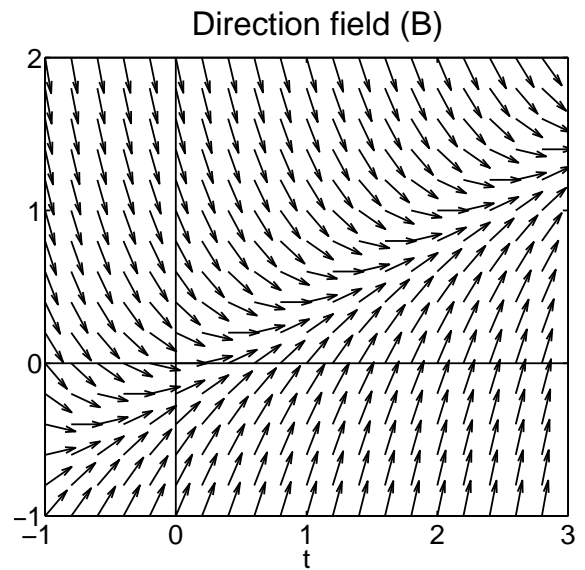
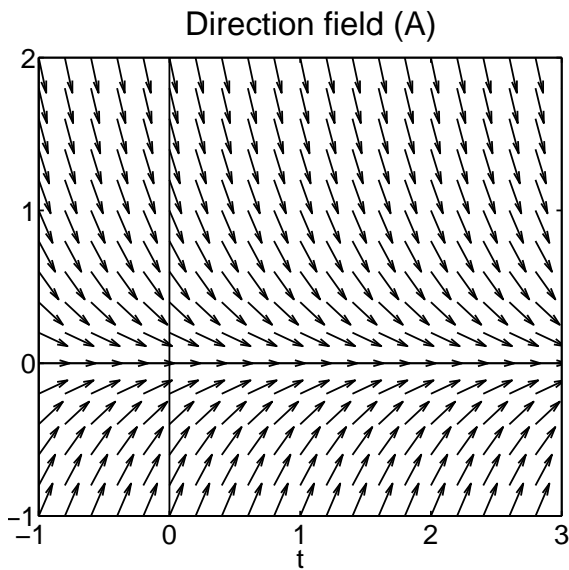
(a) Assuming the hailstone has the shape of a sphere, show that its mass at any given time t is $m(t) = \frac{4\pi}{3}k^3t^3$. (**Hint:** mass = density \times volume)

(b) Substitute for the mass in the equation and solve the IVP for $v(t)$.

(c) Show that the hailstone falls as though it were under one-fourth the influence of gravity, i.e. find the time derivative of $v(t)$ to be $\frac{dv}{dt} = \frac{g}{4}$.

More on the back; turn over the page.

5. (18 points, 6 each) Consider the following direction fields that correspond to three different first order ordinary differential equations:



- (a) Classify the equations corresponding to fields (A) and (B) according to homogeneous or inhomogeneous.
- (b) Classify the equation corresponding to field (C) according to linear or nonlinear.
- (c) Match the following solutions to the direction fields:

(i) $y_1 = \frac{e^t}{e^t - c}$ (ii) $y_2 = ce^{-2t}$ (iii) $y_3 = ce^{-2t} + \frac{t}{2} - \frac{1}{4}$

Good Luck!!!