APPM 2360 Exam 1 Summer 2008

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your lecture section, (4) your instructor's name and (5) a grading table. You have 90 minutes to work all 5 problems on the exam. Each problem is worth 20 points. Show ALL of your work in the bluebook and box in final answers. Start each problem on a new page. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes and calculators are NOT permitted. One letter size ($8.5'' \times 11''$) crib sheet with anything hand written on both sides is allowed.

1. For each of the following differential equations, determine its order, whether it is linear or nonlinear, and if it is linear determine whether it is homogeneous or nonhomogeneous, and whether the coefficients are constant or variable:

   a) $y''' + \sqrt{t}y' = 2t$

   b) $\dfrac{y'}{y} = t$

   c) $y'' + yy' = 0$

   d) $\sqrt{3}y'' + y' + y = 16t$

2. Consider the initial value problem:

$$y' + \left(3t^2 + \frac{1}{t}\right) y = t\,, \qquad y(1) = \frac{4}{3}$$

   a) Determine an integrating factor for the differential equation.

   b) Determine the general solution to the differential equation.

   c) Determine the solution to the initial value problem.

3. Consider the initial value problem:

$$y' = t^3 \mathrm{e}^{t^2 - y}\,, \qquad y(0) = 0$$

   a) What does Picard's Theorem tell us about the initial value problem?

   b) Determine the solution to the initial value problem.

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4. After being on for a long time, the bulb of a lamp has reached a temperature of 140°F. The lamp is turned off and the bulb cools according to Newton's law of cooling in a room of temperature 60°F. Jim wishes to change the bulb but wants to wait until it has cooled to 80°F. If after one minute the bulb has cooled to 136°F, how long from turning off the lamp must Jim wait?

5. Consider the initial value problem:

$$y' = \sqrt{ty}\,, \qquad y(1) = 1$$

a) Solve the initial value problem and determine $y(2)$.

b) Reproduce the following table in your bluebook:

n	t_n	y_n
0		

Using Euler's method:

$$\begin{aligned} t_{n+1} &= t_n + h \\ y_{n+1} &= y_n + hf(t_n, y_n) \end{aligned}$$

fill in the table and find an approximation to $y(2)$. Use $h = \frac{1}{2}$. (You should not need a calculator.)

c) What is the absolute error of this approximation?