(1) a.) 3rd order, linear, Nonhom., var. coef.

b.) 1st order, linear, homo, var. coef

c.) 2nd order, nonlinear.

d.) 2nd order, linear, nonhom, const coeffs

(2) a.) $\mu = e^{\int (3t^2 + \frac{1}{t})dt} = e^{t^3 + \ln t} = te^{t^3}$

b.) $y' + (3t^2 + \frac{1}{t})y = t$

$\rightarrow te^{t^3}y' + (3t^2 + \frac{1}{t})te^{t^3}y = t^2e^{t^3}$

$\rightarrow \frac{d}{dt}[te^{t^3}y] = t^2e^{t^3}$

$\rightarrow te^{t^3}y = \int t^2e^{t^3}dt = \frac{1}{3}e^{t^3} + C$

$\Rightarrow y = \frac{1}{3t} + \frac{Ce^{-t^3}}{t}$

c.) $y(1) = \frac{4}{3} = \frac{1}{3} + \frac{Ce^{-1}}{1} \Rightarrow 1 = Ce^{-1} \Rightarrow C = e$

$\rightarrow y(t) = \frac{1}{t}(\frac{1}{3} + e^{(1-t^3)})$

③ $y' = t^3 e^{t^2} e^{-y} \qquad y(0)=0$

10 a.) $f = t^3 e^{t^3} e^{-y}$

$f_y = -t^3 e^{t^2} e^{-y}$ 4pts

Since f and f_y are continuous $\forall t$ and $\forall y$, by Picard's Thm we know: $y' = t^3 e^{t^2} e^{-y}$, $y(0)=0$ has a unique soln (4pts) for t in the interval $(-h, h)$ ~~for~~ some $h \in \mathbb{R}$. (2pts)

10 b.) $\int e^y dy = \int t^3 e^{t^2} dt = \int t^2 (t e^{t^2}) dt = \frac{t^2}{2} e^{t^2} - \int t e^{t^2}$

($t^2 \to 2t$, $t e^{t^2} \to \frac{1}{2} e^{t^2}$)

$\to e^y = \frac{t^2}{2} e^{t^2} - \frac{1}{2} e^{t^2} = e^{t^2}\left(\frac{t^2}{2} - \frac{1}{2}\right) + C$ 5pts

$\to \quad y = \ln\left(\frac{e^{t^2}}{2}(t^2 - 1) + C\right)$

$y(0) = 0 = \ln\left(\frac{1}{2}(-1) + C\right)$

$\Rightarrow 1 = -\frac{1}{2} + C \Rightarrow C = \frac{3}{2}$ ← 2pts

$\to y = \ln\left(\frac{e^{t^2}}{2}(t^2 - 1) + \frac{3}{2}\right)$ ← 1pt

Separate: 2pts.

(4) $T(0) = 140°F$, $T(1) = 136$

$$\frac{dT}{dt} = k(M - T) = k(60 - T)$$

$$\frac{dT}{dt} + kT = 60k$$

$$\mu = e^{kt}$$

$$\frac{d}{dt}\left[e^{kt}T\right] = 60ke^{kt}$$

$$e^{kt}T = 60e^{kt} + C$$

$$T = 60 + Ce^{-kt}$$

$$T(0) = 140 = 60 + C \Rightarrow C = 80$$

$$T = 20(3 + 4e^{-kt})$$

$$T(1) = 136 = 60 + 80e^{-k(1)}$$

$$-\ln\left(\frac{76}{80}\right) = k = -\ln\left(\frac{19}{20}\right)$$

$$\rightarrow T(t) = 20\left(3 + 4e^{\ln\left(\frac{19}{20}\right)t}\right)$$

$$80 = 20\left(3 + 4e^{\ln\left(\frac{19}{20}\right)t}\right)$$

$$\frac{\ln\frac{1}{4}}{\ln\frac{19}{20}} = t = \frac{\ln 4}{\ln\left(\frac{20}{19}\right)} \text{ min} \approx 27 \text{ min.}$$

5.) a) $y' = \sqrt{ty}$

8 pts

$$\frac{dy}{\sqrt{y}} = \sqrt{t}\, dt \quad 2 \text{ pts}$$

$$2y^{\frac{1}{2}} = \frac{2}{3}t^{3/2} + C \quad 3 \text{ pts}$$

$$y = \left(\frac{1}{3}t^{3/2} + C_1\right)^2$$

$$y(1) = 1 = \left(\frac{1}{3} + C_1\right)^2 \Rightarrow C_1 = \frac{2}{3} \quad 1 \text{ pt}$$

$$y(t) = \left(\frac{1}{3}t^{3/2} + \frac{2}{3}\right)^2 = \frac{1}{9}\left(t^{3/2} + 2\right)^2$$

$$= \frac{1}{9}\left(t^3 + 4t^{3/2} + 4\right) \quad 1 \text{ pt.}$$

$$y(2) = \frac{1}{9}\left(8 + 4\sqrt{8} + 4\right) = \frac{1}{9}\left(12 + 8\sqrt{2}\right) \quad 1 \text{ pt.}$$

8 pts b.)

$t_0 = 1$

$y_0 = 1$

$t_1 = 3/2$

$y_1 = 1 + \frac{1}{2}(\sqrt{1 \cdot 1}) = 3/2$

$t_2 = 2$

$y_2 = 3/2 + \frac{1}{2}\sqrt{\frac{3}{2} \cdot \frac{3}{2}} = 3/2 + 3/4 = 9/4$

n	t_n	y_n
0	1	1
1	3/2	3/2
2	2	9/4

4 pts. c.) $\text{Error} = \left| \frac{9}{4} - \frac{1}{9}(12 + 8\sqrt{2}) \right|$

$$= \left| \frac{81 - 48}{36} - \frac{8}{9}\sqrt{2} \right| = \left| \frac{11}{12} - \frac{8}{9}\sqrt{2} \right| = \frac{1}{3}\left| \frac{11}{4} - \frac{8}{3}\sqrt{2} \right|$$

$$\approx \frac{1}{3}\left(\frac{8}{3}\sqrt{2} - \frac{11}{4}\right)$$