APPM 2360 Exam 2 Summer 2008

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your lecture section, (4) your instructor's name and (5) a grading table. You have 90 minutes to work all 5 problems on the exam. Each problem is worth 20 points. Show ALL of your work in the bluebook and $\boxed{\text{box}}$ in final answers. Start each problem on a new page. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes and calculators are NOT permitted. One letter size (8.5″ × 11″) crib sheet with anything hand written on both sides is allowed.

1. a) Given the matrices $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & -3 \\ 0 & 4 & 4 \end{bmatrix}$, which of the following matrix operations are defined and which are not defined? (You do not need to give reasons or evaluate those that are defined.)

   i) AB

   ii) A^2

   iii) $A^{\mathsf{T}}B$

   iv) BB^{T}

   v) $A + B^{\mathsf{T}}$

   b) Determine all values of a and b for which the matrix $C = \begin{bmatrix} 3 & a \\ 0 & b \end{bmatrix}$ has only one eigenvector.

   c) Give an example of three 2×2 matrices D, E and F, with $E \neq F$ but $DE = DF$.

   d) Prove there does not exist a matrix G, with $G^2 = \begin{bmatrix} 2 & -3 \\ 6 & -1 \end{bmatrix}$ and $G^3 = \begin{bmatrix} -2 & -5 \\ 8 & -7 \end{bmatrix}$. (hint: think determinants)

2. Consider the matrix, $A(t) = \begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix}$.

   a) Compute the determinant of $A(t)$.

   b) Use your answer from part (a) to answer the following. Give reasons.

   i) Are $\cos(t)$ and $\sin(t)$ linearly independent functions?

   ii) Are there any values of t for which the columns of $A(t)$ do not span $\mathbb{R}^2$?

   c) Compute the eigenvalues of $A(t)$.

   d) Compute the eigenvectors of $A(\frac{\pi}{2})$.

— over —

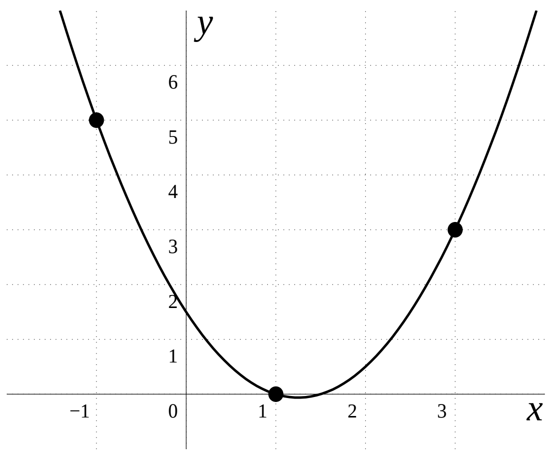
3. In this problem you will determine the equation of a parabola that passes through the points

$$\begin{array}{lll} \text{i)} & x=-1, & y=5, \\ \text{ii)} & x=1, & y=0, \\ \text{iii)} & x=3, & y=3, \end{array}$$

as shown in the figure. Assume that the equation of the parabola is

$$y = ax^2 + bx + c \qquad (1)$$

where a, b and c are unknown coefficients.



a) Substitute each (x, y) point into (1) to obtain 3 algebraic equations that relate the 3 unknowns a, b and c.

b) Rewrite the 3 algebraic equations determined in (a) as a single augmented matrix and row reduce this matrix to RREF (reduced row echelon form).

c) Using your answer from (b) determine the values of a, b and c and write the equation of the parabola.

4. Let V be a vector space and W a subset of V.

a) State the 2 closure properties that W must satisfy to be a linear subspace.

b) Let $M_{2,2}$ denote the vector space of all 2×2 matrices.
Let $C([0,1])$ denote the vector space of all continuous functions on the interval $[0,1]$.
Are the following linear subspaces? If yes, simply write "YES" (no justification required). If no, write "NO" and provide reasoning.

i) $W_1 = \{A \in M_{2,2} \mid A \text{ is a diagonal matrix}\}$

ii) $W_2 = \{A \in M_{2,2} \mid \det(A) = 0\}$

iii) $W_3 = \{f \in C([0,1]) \mid f(0) = 1\}$

5. Determine all eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 3 & 4 & 5 \end{bmatrix}$.