APPM 2360 Exam 3 Summer 2008

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your lecture section, (4) your instructor's name and (5) a grading table. You have 90 minutes to work all 4 problems on the exam. Each problem is worth 25 points. Show ALL of your work in the bluebook and box in final answers. Start each problem on a new page. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes and calculators are NOT permitted. One letter size ($8.5'' \times 11''$) crib sheet with anything hand written on both sides is allowed.

1. a) Is the differential equation, $\ddot{\theta} + \sin(\theta) = 0$, conservative or non-conservative? Give a reason.

   b) An undamped, unforced, mass-spring system is modelled by $m\ddot{x} + kx = 0$. If $m = 9\ kg$ and the period of oscillations is $T = 6\ s$, determine the spring constant, k. Give units.

   c) Convert the one-dimensional, third order, differential equation, $\dddot{x} + 5\dot{x} = x^3$, to a three-dimensional, first order system.

   d) Determine the general solution to $\ddot{x} + 2a\dot{x} + a^2x = 0$, where a is any real number.

   e) Suppose $y_1(t) = \frac{1}{2}t^2$ and $y_2(t) = t^2 + 2$ are solutions to the differential equation, $\ddot{y} + q(t)y = f(t)$. Determine $f(t)$.

2. Consider the following differential equation:

$$y'' + 2y' - 3y = f(t)\ .$$

   a) Determine the homogeneous solution.

   b) For each of the following forcing functions write down the form of the particular solution according to the method of undetermined coefficients. If this is not possible write "NOT". You do not need to find the values of the coefficients.

      i) $f(t) = t^2 - 3$
      ii) $f(t) = \mathrm{e}^{t^2}$
      iii) $f(t) = \sin(t) + \cos(5t)$
      iv) $f(t) = t^3\mathrm{e}^{-3t}$
      v) $f(t) = 4\mathrm{e}^{-3t}\sin(t)$

   c) Using $f(t) = 4\mathrm{e}^{-3t}\sin(t)$, determine the particular solution of the differential equation using variation of parameters.

   d) Write the general solution to the differential equation when $f(t) = 4\mathrm{e}^{-3t}\sin(t)$.

3. A damped, unforced, mass-spring system with $m = 1\ kg$, $b = 3\ kg\,s^{-1}$ and $k = 2\ kg\,s^{-2}$ is described by

$$\ddot{x} + 3\dot{x} + 2x = 0$$

and has initial conditions

$$\begin{aligned} x(0) &= 0\ m \\ \dot{x}(0) &= 6\ m\,s^{-1}\ . \end{aligned}$$

a) Solve this IVP.

b) Determine the time, T, at which x obtains its maximum value.

c) Determine the amount of energy lost from $t = 0$ to $t = T$.

4. Consider a forced, undamped, mass-spring system with a mass of 2 kg, spring constant $50\ N\,m^{-1}$ and forcing function $f(t) = -20\sin(5t)$.

a) Write down the differential equation that describes this system.

b) Determine the homogeneous solution.

c) Determine a particular solution using the method of undetermined coefficients.

d) Suppose initially the spring is held 1 m to the left (negative x direction) and given a velocity of $1\ m\,s^{-1}$ to the right. Determine the solution to this IVP.

e) What happens as $t \to \infty$, and what word do we use to describe this type of dynamical behavior?