

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and electronic devices of ANY sort are NOT permitted. One $8'' \times 11''$, two-sided, sheet of notes is allowed.

1. (20 points) Solve the following Initial Value Problems.

(a) $\frac{dy}{dt} - y = e^{3t}$, $y(0) = 1$.

(b) $t\frac{dy}{dt} + 3(t+1)y = \frac{1}{t^2}$, $y(1) = 0$.

2. (20 points) Short answer questions. Explain your answer (a correct answer with no justification may receive no credit).

(a) If L is a linear operator, and $y(t)$ is a function such that $L(y(t)) = 2e^t$. Then, what is $L(2y(t))$? What is $L(0)$?

(b) If L is a linear operator, and $L(y(t)) = 2e^t$ and $L(z(t)) = 0$, then what is $L(y(t) + 2z(t))$?

(c) Does Picard's theorem apply to the initial value problem

$$(1 - y^2)\frac{dy}{dt} = \cos y, \quad y(0) = 0 ?$$

If so, what does it say. If not, why not?

(d) Verify that the function $y(t) = t^{\frac{3}{2}} + 1$ is a solution to the initial value problem

$$\frac{dy}{dt} = \frac{3}{2}(y-1)^{\frac{1}{3}}, \quad y(0) = 1.$$

Is this the only solution to this IVP? Why or why not? If there is another solution, find it.

3. (20 points) Lehman buys a bundle of subprime mortgage securities for \$1 Million that Wachovia assures him will pay 10% interest per year (continuously compounded) for 10 years. As we all know now, it didn't quite work out that way.

(a) Supposing that it had, how much money would Lehman have at the end of the 10 years? (Since you do not have a calculator, you may leave your answer in terms of functions, for example $3 \cos(\ln(4))$).

(b) What actually happened: each year, \$100,000 worth of the mortgages default (that is, \$100,000/year is continuously removed from Lehman's account). The remaining mortgages continue to perform (that is, pay 10% per year). How much money does Lehman actually have at the end of 10 years?

PLEASE TURN OVER

4. (20 points) Consider the differential equation $t^3 \frac{dy}{dt} = 2y^2$

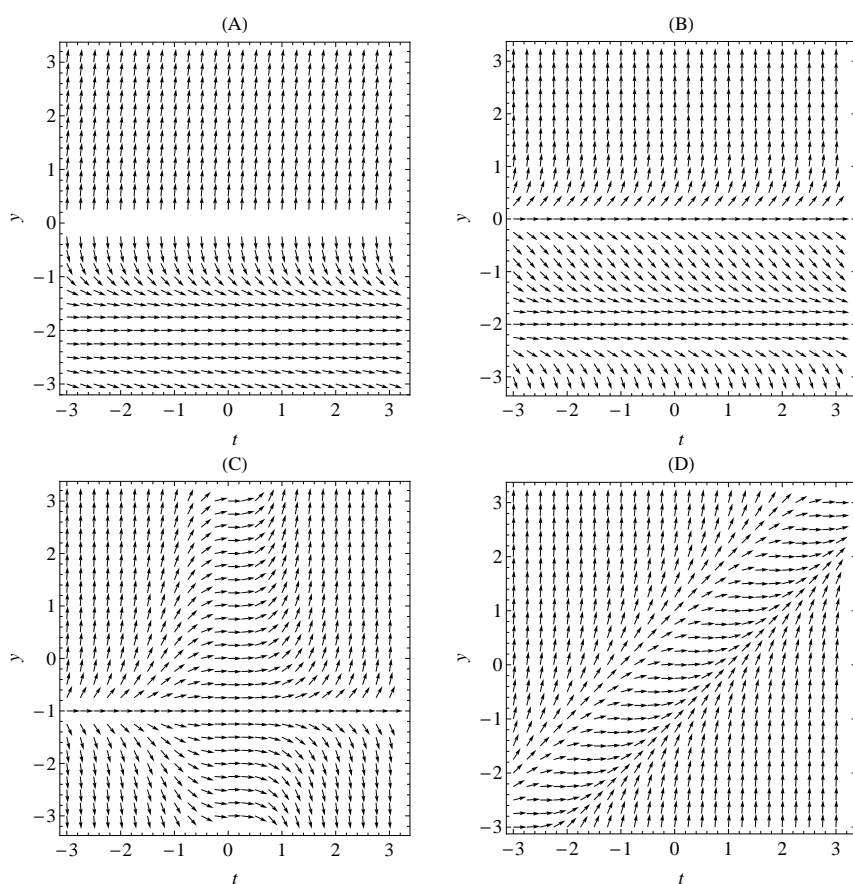
- (a) Find the general solution of the equation.
- (b) Find the solution with the initial condition $y(1) = 1$.
- (c) Using Euler's method

$$t_{n+1} = t_n + h$$

$$y_{n+1} = y_n + hf(t_n, y_n),$$

find the approximation to $y(2)$ with $h = 1/2$ and the initial condition $y(1) = 1$.

5. (20 points) Consider the following direction fields that correspond to four different first order ordinary differential equations:



(a) Match the following differential equations to the direction fields. Explain your answers.

(1) $\frac{dy}{dt} = y(2 + y)^2$, (2) $\frac{dy}{dt} = (y - t)^2$, (3) $\frac{dy}{dt} = (y + 1)t^2$, (4) $y \frac{dy}{dt} = (y + 2)^2$

(b) Classify the equations in part (a) according to linear or nonlinear, autonomous or nonautonomous.

(c) Identify the equilibria shown in *each* of the above graphs and give their stability.