ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Start each problem on a new page. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Books, class notes, calculators and electronic devices of ANY sort are NOT permitted. One $8'' \times 11''$, two-sided, sheet of notes is allowed.

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1. (20 points) True/False and Short Answer questions. No explanations for the answers to this question are needed.
   (a) If $x(t)$ is a solution to the equation $\ddot{x} + 3\dot{x} + 3x = 0$, then $x(t) \to 0$ as $t \to \infty$.
   (b) If all eigenvalues of A are distinct, then A is invertible.
   (c) The eigenvalues of A are the the same as those of A^T.
   (d) If the RREF of a 3×3 matrix A has a zero row, then A has a zero eigenvalue.
   (e) If all the eigenvalues of an $n \times n$ matrix A are positive, then A has n independent eigenvectors.
   (f) What is the natural frequency of a mass $m = 27$ kg on the end of a spring with restoring constant $k = 1$ nt/m and damping constant $b = 2$ nt s/m? If the spring is forced with $f(t) = 13\cos(\omega_f t)$ nt, for what value of ω_f will the forced response have maximum amplitude?

2. (20 points) Consider the matrix

$$A = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & -3 \end{bmatrix} . \tag{1}$$

   (a) Find the eigenvalues of A.
   (b) Find the eigenvectors of A, the corresponding eigenspaces, and their dimensions.
   (c) Find the general solution of the system of differential equations $\mathbf{x}' = A\mathbf{x}$ for a 3-dimensional vector $\mathbf{x}$ and A given in (1).

3. (20 points) Suppose that

$$\ddot{x} - 2\dot{x} + 5x = f(t) .$$

   (a) Find the homogeneous solution, $x_h(t)$.
   (b) Using the method of undetermined coefficients, find a particular solution $x_p(t)$ when $f(t) = 25t^2$.
   (c) Using the method of undetermined coefficients, write out the **form** of the particular solution $x_p(t)$ when $f(t) = t\cos(2t)$. Do **not solve** for any constants, just write down the form of x_p (for example "$x_p(t) = \alpha t + \beta$, where α and β are constants").

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4. (20 points) Consider the second order equation:

$$t^2y'' - 2ty' + 2y = t^3 \sin t \ .$$

   (a) Show that $y_1(t) = t$ and $y_2(t) = t^2$ are linearly independent, and that they are solutions of the corresponding homogeneous equation, $t^2y'' - 2ty' + 2y = 0$.

   (b) Use variation of parameters to find the general solution for the original nonhomogeneous equation.

5. (20 points) Consider the system of equations:

$$\begin{cases} \dot{x} = & x + 2y \\ \dot{y} = & 2x + y \end{cases} .$$

   (a) Find the general solution.

   (b) Find the solution with initial values $x(0) = 1$, $y(0) = 0$.

   (c) Sketch the phase portrait for the system.