#1

a) True
b) False
c) True
d) True
e) False
f) $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{1}{27}}$

$$\omega_{peak} = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}} = \sqrt{\frac{1}{27} - \frac{4}{2 \cdot 27^2}} = \sqrt{\frac{27-2}{27^2}} = \frac{5}{27}$$

T a) $b > 0 \Rightarrow$ damping yes

F b) could have $\lambda = 0$ & still be distinct.

e.g. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

T c) $\det(A - \lambda I) = \det(A^T - \lambda \underset{\uparrow \atop I = I^T}{I^T}) = \det(A^T - \lambda I)$

T d) yes, $\det(A) = 0$ so $\det(A - 0I) = 0$

F e) no e.g. $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ only one vector

#2 a) $(1-\lambda)\begin{bmatrix} -1-\lambda & -1 \\ 1 & -3-\lambda \end{bmatrix} = (1-\lambda)\left[\lambda^2+4\lambda+4\right]$

$\boxed{\lambda=1, \quad \lambda = -2 \pm \sqrt{4-4} = -2}$ (double)

b) $\lambda=1$ $\begin{bmatrix} -2 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -4 \end{bmatrix} v_1 = 0 \Rightarrow v_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda=-2$ $\begin{bmatrix} 1 & 0 & -1 \\ 0 & +3 & 0 \\ 1 & 0 & -1 \end{bmatrix} v_2 = 0 \Rightarrow v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

deficiency
only one eigenvector

c) Need generalized eigenvector

$(A-\lambda I)u = v_2$

$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 3 & 0 & 0 \\ 1 & 0 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow u = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

any c okay.

Ex $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ or $u = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ etc.

Solution $\boxed{x(t) = c_1 e^{t}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 e^{-2t}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_3 e^{-2t}\left(t\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)}$

3) $r^2-2r+5=0 \Rightarrow (r-\ )(r-\ )$

$r = 1 \pm \sqrt{1-5} = 1 \pm 2i$

a) $\boxed{X_h = e^t\left(c_1\cos 2t + c_2 \sin 2t\right)}$

b) $S' = \{t^2, t, 1\} \Rightarrow x_p = At^2 + Bt + C$

$f(t) = 25t^2$

	t^2	t	1
$5x_p$	$5A$	$5B$	$5C$
$-2x_p'$	0	$-4A$	$-2B$
x_p''	0	0	$+2A$
f	25	0	0

$A=5$ $\quad 5B = 4A = 20$

$5C = 2B - 2A = -2$

$C = -2/5$

$\boxed{x_p = 5t^2 + 4t - 2/5}$

c) $f(t) = t\cos 2t$

$S = \{t\cos 2t,\ t\sin 2t,\ \cos 2t,\ \sin 2t\}$ ← not part of x_h ...

$\boxed{x_p = At\cos 2t + Bt\sin 2t + C\cos 2t + D\sin 2t}$

4)

a) $y_1 = t$ $\quad t^2(0) - 2t(1) + 2t = 0$ ✓

$y_2 = t^2$ $\quad t^2(2) - 2t(2t) + 2t^2 = (2-4+2)t^2 = 0$ ✓

~~$y_p = v_1 t$~~

Show independent: $W(y_1, y_2) = \det\begin{bmatrix} t & t^2 \\ 1 & 2t \end{bmatrix} = t^2 \neq 0.$

QED

b) $y_p = v_1 t + v_2 t^2$ $\quad f = t\sin t \Leftarrow$ note Must put in Std form!!

$$v_1' = -\frac{y_2 f}{W} = -\frac{t^2}{t^2} t\sin t$$

$$v_1' = -t\sin t$$

$$v_1 = -\int t\sin t\,dt = t\cos t - \int \cos t$$

$u = -t \quad du = -dt$

$dv = \sin t \quad v = -\cos t$

$$= t\cos t - \sin t$$

$$v_2' = +\frac{y_1 f}{W} = \frac{t}{t^2} t\sin t = \sin t$$

$$\Rightarrow v_2 = -\cos t$$

So

$$y = y_h + y_p$$

$$= c_1 t + c_2 t^2 + \cancel{t^2\cos t} - t\sin t - \cancel{t^2\cos t}$$

$$\boxed{y = c_1 t + c_2 t^2 - t\sin t.}$$

5) $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $\quad \lambda^2 - 2\lambda - 3 = (\lambda+1)(\lambda-3)$

$\lambda = -1 \quad \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} v_1 = 0 \Rightarrow v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda = 3 \quad \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} v_2 = 0 \Rightarrow v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

a) $\boxed{\underset{\sim}{x} = c_1 e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

b) $\underset{\sim}{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ -1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 2 & 1 \end{array}\right] \quad \begin{array}{l} c_1 = 1/2 \\ \Rightarrow c_2 = 1/2 \end{array}$

$\underset{\sim}{x}(t) = \frac{1}{2} e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{1}{2} e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \boxed{\frac{1}{2} \begin{bmatrix} e^{-t} + e^{3t} \\ -e^{-t} + e^{3t} \end{bmatrix}}$

c)