APPM 2360 Exam 1 Summer 2009

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your lecture section, (4) your instructor's name and (5) a grading table. You have 90 minutes to work all 5 problems on the exam. Each problem is worth 20 points. Show ALL of your work in the bluebook and box in final answers. Start each problem on a new page. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes and calculators are NOT permitted. One letter size ($8.5'' \times 11''$) crib sheet with anything hand written on both sides is allowed.

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1. Consider the Initial Value Problem

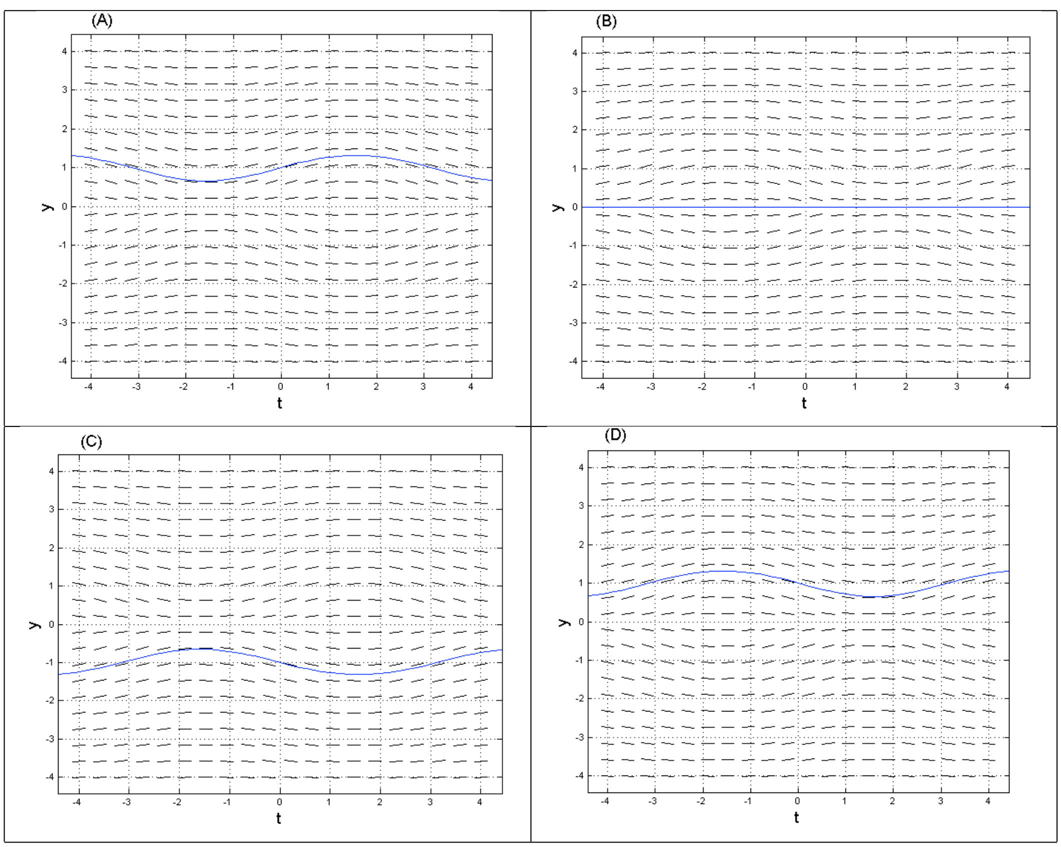
$$y' = \frac{y \cos t}{1 + 2y^2}, \quad y(0) = 1$$

   (a) Algebraically find the equilibrium solution(s) to this differential equation.

   (b) Use *separation of variables* to find the family of solutions to this differential equation. (Leave you answer in implicit form).

   (c) Find the specific solution to the IVP.

   (d) Which of the following direction fields corresponds to the above differential equation and solution to the IVP?



2. Use Euler's method to approximate the solution to the Initial Value Problem $y' = t - y$, $y(0) = 1$ on the interval [0,2], with step size $h = 1$

3. Consider the differential equation $(1 + t^2)y' = 2ty + (1 + t^2)^2$.

   (a) Put the equation in standard form and write down the integrating factor.

   (b) Find the homogeneous solution $y_h(t)$ to the differential equation.

   (c) Using *variation of parameters*, find the particular solution $y_p(t)$ to the differential equation.

   (d) Find the specific solution if $y(1) = 0$.

4. You are at a party at your best friend's apartment when you see someone pour 1 Liter of vodka into the punch bowl which contained 3 Liters of pure punch. Being a good friend you immediately start pouring in pure punch at a rate of 1 Liter per minute and pull the drain on the punch bowl so that it drains at a rate of 1 Liter per minute. You stir the punch continuously so that the draining liquid is well mixed.

   (a) Write down the Initial Value Problem representing this situation, using $v(t)$ as the amount of vodka in the bowl at time t.

   (b) Solve the IVP from part (a).

   (c) How long must you continue in order to reduce the amount of vodka to $\frac{1}{4}$ Liter?

5. Consider the Initial Value Problem

$$y' = y^{1/3}, \quad y(0) = 0, \quad t \geq 0$$

   (a) Verify that both $y_1 = (\frac{2}{3}t)^{3/2}$ and $y_2 = -(\frac{2}{3}t)^{3/2}$ are solutions to the above IVP.

   (b) Use Picard's Theorem to explain why this IVP does not necessarily have a unique solution.