Problem 1

a.) $y' = \frac{y \cos t}{1+2y^2}$

Find $y' = 0 = y \cos t \Rightarrow \boxed{y=0}$

b.) $\frac{(1+2y^2)dy}{y} = \cos(t)\,dt \Rightarrow \int\left(\frac{1}{y} + 2y\right)dy = \int \cos t \, dt$

$\Rightarrow \boxed{\ln|y| + y^2 = \sin t + C}$

c.) $y(0) = 1 \Rightarrow \ln|1| + 1 = \sin(0) + C$

$\Rightarrow C = 1$

$\Rightarrow \boxed{\ln|y| + y^2 = \sin t + 1}$

d.) $\boxed{A}$

Problem 2

$t_0 = 0$

$y_0 = 1$

$t_1 = 0 + 1 = 1$

$y_1 = 1 + (0-1)\cdot 1 = 0 \Rightarrow y(1) \approx 0$

$t_2 = 1+1 = 2$

$y_2 = 0 + (1-0)\cdot 1 = 1 \Rightarrow y(2) \approx 1$

Problem 3

a) $$\boxed{y' - \frac{2t}{(1+t^2)}\, y = 1 + t^2}$$

$$\mu(t) = e^{\int \frac{-2t}{(1+t^2)}\,dt} = e^{-\ln|1+t^2|} = \boxed{\frac{1}{1+t^2} = \mu(t)}$$

b) $$y' = \frac{2t}{1+t^2}\, y$$

$$\Rightarrow \frac{dy}{y} = \frac{2t}{1+t^2}\,dt \Rightarrow \ln|y| = \ln|1+t^2| + C$$

$$\Rightarrow y_h = C(1+t^2)$$

c) Guess $y_p(t) = V(t)\cdot(1+t^2)$

$$\Rightarrow y_p' = V'(1+t^2) + V\cdot 2t$$

$$\Rightarrow y_p' - \frac{2t}{1+t^2}\, y_p = V'(1+t^2) + \cancel{V\cdot 2t} - \cancel{\frac{2t}{1+t^2}\cdot V\cdot(1+t^2)}$$

$$\Rightarrow V'(1+t^2) = 1 + t^2$$

$$\Rightarrow V' = 1 \Rightarrow V = t$$

$$\Rightarrow y_p(t) = t(1+t^2)$$

d) $$y_G = y_p + y_h = t(1+t^2) + C(1+t^2)$$

$$y(1) = 0 \Rightarrow 0 = 2 + 2C \Rightarrow C = -1$$

$$\Rightarrow \boxed{y(t) = t + t^3 - 1 - t^2}$$

Problem 4

a.) $\frac{dv}{dt} = 1\,^{L}/_{min} \cdot 0\,^{L}/_{L} - 1\,^{L}/_{min}\,\frac{V}{4}\,^{L}/_{L}$

$$\boxed{\frac{dv}{dt} = -V/4\,, \quad V(0) = 1}$$

b.) $V(t) = 1e^{-\frac{1}{4}t}$

c.) $\frac{1}{4} = e^{-\frac{1}{4}\cdot t} \Rightarrow t = -4\ln\frac{1}{4} = \boxed{4\ln(4)\ \text{min}}$

≈ 5.5 min

Problem 5

a.) $y_1 = (\frac{2}{3}t)^{3/2} \Rightarrow LHS = y_1' = \cancel{\frac{3}{2}}(\frac{2}{3}t)^{\frac{1}{2}} \cdot \cancel{\frac{2}{3}}$

$= \sqrt{\frac{2}{3}t}$

$RHS = (y_1)^{1/3} = ((\frac{2}{3}t)^{3/2})^{\frac{1}{3}} = \sqrt{\frac{2}{3}t}$ ✓

And $y_1(0) = (\frac{2}{3}\cdot 0)^{3/2} = 0$

$y_2 = -(\frac{2}{3}t)^{3/2} \Rightarrow LHS = y_2' = -\cancel{\frac{3}{2}}(\frac{2}{3}t)^{\frac{1}{2}} \cdot \cancel{\frac{2}{3}} = -\sqrt{\frac{2}{3}t}$

$RHS = (y_2)^{\frac{1}{3}} = (-(\frac{2}{3}t)^{3/2})^{\frac{1}{3}} = -\sqrt{\frac{2}{3}t}$ ✓

And $y_2(0) = -(\frac{2}{3}\cdot 0)^{3/2} = 0$

b.) Well $f(t,y) = y^{1/3}$ is continuous for all t and y so at least one solution exists through any (t_0, y_0), but $f_y = \frac{1}{3}y^{-1/3}$ is not continuous through $y=0$ so for the given initial value $y(0)=0$ we cannot guarentee a unique solution