# APPM 2360 Exam 1 Summer 2009

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your lecture section, (4) your instructor's name and (5) a grading table. You have 90 minutes to work all 5 problems on the exam. Each problem is worth 30 points. Show ALL of your work in the bluebook and box in final answers. Start each problem on a new page. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes and calculators are NOT permitted. One letter size ($8.5'' \times 11''$) crib sheet with anything hand written on both sides is allowed.

1. (30 points) For each of the following, determine if $W$ is a subspace of V or not. **Answer YES or NO without justification.**
   (a) $V = \mathbb{R}^2$, $W = \{(x, y) | x = 0\}$
   (b) $V = \mathbb{M}_{22}$, $W =$ the set of all 2×2 matrices with determinant equal to zero.
   (c) $V = \mathbb{P}_2$, $W = \{t, t^2 + 1, 3\}$
   (d) $V = C[0, 1]$, $W =$ the set of all continuous functions, $f$, defined on [0, 1], such that $f(0) = 1$.
   (e) $V = C(-\infty, \infty)$, $W =$ the set of solutions to $y' + 3y = 0$.

2. (30 points) Recall that the trace of a matrix $\mathbf{A}$, denoted Tr$\mathbf{A}$, is the *sum* of the diagonal entries of $\mathbf{A}$:
$$\text{Tr}\mathbf{A} = a_{11} + a_{22} + ... + a_{nn} = \sum_{k=1}^{n} a_{kk}$$
   (a) Show that $\text{Tr}(\mathbf{A} + \mathbf{B}) = \text{Tr}\mathbf{A} + \text{Tr}\mathbf{B}$
   (b) Show that $\text{Tr}(c\mathbf{A}) = c\text{Tr}\mathbf{A}$
   (c) Let $W$ be the set of $2 \times 2$ diagonal matrices with entries in $\mathbb{R}$ and trace 0. Show that $W = \left\{ \begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix}, a \in \mathbb{R} \right\}$ is a subspace of $\mathbb{M}_{22}$.
   (d) Exhibit a basis for $W$ and give the dimension of this subspace.

3. (30 points) Consider the following linear system of 3 equations and 3 unknowns, where $b \neq 0$.
$$\begin{aligned} x_1 + x_2 + x_3 &= b \\ x_1 + x_2 + x_3 &= b \\ x_1 + x_2 + x_3 &= b \end{aligned}$$
   (a) Use Gauss-Jordan reduction to transform the augmented matrix of this system to RREF. *Show all your steps. Be sure to write down what row operations you are using.*
   (b) How many pivots does the coefficient matrix $\mathbf{A}$ have? What is the rank of $\mathbf{A}$? How many solutions does this linear system of equations have?
   (c) Determine the solution set for this linear system. Is the solution set a **vector space**? Why or why not.

4. (30 points) Suppose $A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.

   (a) Find $A^{-1}$.

   (b) Use $A^{-1}$ to solve the linear system $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

   (c) Verify your solution to part (b) using Cramer's Rule. Show all your steps.

5. (30 points)

   (a) Write down the standard equation which relates a matrix $A$ to its eigenvalue $\lambda$ and eigenvector $\mathbf{v}$.

   (b) Find the eigevalues of $A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & -3 & -1 \\ 0 & 2 & -1 \end{bmatrix}$.

   (c) Find the eigevectors associated with each eigenvalue of $A$.