APPM 2360 Exam 3 Summer 2009

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your lecture section, (4) your instructor's name and (5) a grading table. You have 90 minutes to work all 4 problems on the exam. The point values are indicated at the start of each problem. Show ALL of your work in the bluebook and box in final answers. Start each problem on a new page. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes and calculators are NOT permitted. One letter size ($8.5'' \times 11''$) crib sheet with anything hand written on both sides is allowed.

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1. (30 points) For each of the following differential equations write down the predicted form for the particular solution, $y_p(t)$, to be used in the method of undetermined coefficients. **Do not solve for the coefficients A, B, C etc.**
   i. $y'' + y = e^{-t} + t^3 - 3$
   ii. $y'' - 2y + 1 = 2t\cos 4t$
   iii. $y'' + 2y' - y = -e^{-t}sin4t$
   iv. $y'' - y = 2te^{-t}$

2. (40 points) Consider the following 2nd order, linear, non-homogeneous differential equation.

$$y'' - 3y' + 2y = 2te^{-t}$$

   (a) Find the solution $y_h(t)$ to the corresponding homogeneous differential equation.

   (b) Find the particular solution $y_p(t)$ to the non-homogeneous differential equation using the **method of undetermined coefficients**.

   (c) Find (again) the particular solution $y_p(t)$ to the non-homogeneous differential equation using **variation of parameters**.

   (d) Write down the general solution to the original differential equation.

   (e) Convert the **homogeneous** equation to a system of two first order equations.

   (f) Determine the equlibria and nullclines of this system.

— **over** —

3. (40 points) Consider the forced, undamped mass-spring system with a mass of 1 kg, spring constant $144\pi^2\ Nm^{-1}$ and forcing function $f(t) = 11\cos(10\pi t)$.

   (a) Write down the differential equation that describes this system.

   (b) Determine the homogeneous solution.

   (c) Determine a particular solution any way you know how.

   (d) Suppose that initially the system is at rest so the initial conditions are $\dot{x}(0) = 0$ and $x(0) = 0$. Find the solution to this initial value problem. You may wish use the following trig identity:

$$\cos u - \cos v = 2\sin\left(\frac{v-u}{2}\right)\sin\left(\frac{u+v}{2}\right)$$

   (e) What word do we use to describe the behavior of this system? Draw a sketch of the solution in the (t,x)-plane.

4. (40 points) So far in our investigation of higher order DE's we have developed techniques that allow us to solve **linear** DE's. For this problem, however, consider the following **nonlinear** initial value problem:

$$(\star) \qquad x'' + \frac{1}{(x-2)^3} = 0 \qquad x(0) = 0,\ x'(0) = \frac{1}{2}$$

   Follow steps (a) - (e) to obtain a solution to this IVP:

   (a) Using the substitution, $v = x' = \frac{dx}{dt}$, rewrite $(\star)$ as a first order differential equation containing only v and x. (Hint: The chain rule gives $v' = \frac{dv}{dt} = \frac{dx}{dt}\frac{dv}{dx}$.)

   (b) Solve the equation you found in (a), using the given initial values to solve for the constant of integration. (Hint: Choose minus when you take the square root.)

   (c) Substitute x' back in for v and solve this new first order differential equation. Use the initial values to solve for the constant of integration. You may leave to solution in implicit form.

   (d) Find the potential energy of the conservative system defined by $(\star)$.

   (e) Find the total energy $E(x, \dot{x})$. Use this to find the equilibrium points of the system.