Problem 1

i.) $y_p(t) = Ae^{-t} + B_3 t^3 + B_2 t^2 + B_1 t + B_0$

ii.) $y_p(t) = (At + B)\cos(4t) + (Ct + D)\sin(4t)$

iii.) $y_p(t) = Ae^{-t}\cos(4t) + Be^{-t}\sin(4t)$

iv.) $y_p(t) = t(At + B)e^{-t}$

Problem 2

a.) $y_h'' - 3y_h' + 2y_h = 0$

$r^2 - 3r + 2 = 0 \Rightarrow (r-2)(r-1) = 0 \Rightarrow r_1 = 2,\ r_2 = 1$

$\Rightarrow \boxed{y_h(t) = c_1 e^{2t} + c_2 e^{t}}$

b.) $y_p(t) = (At + B)e^{-t}$

$\Rightarrow y_p'(t) = -e^{-t}(At + B) + e^{-t}(A) = e^{-t}(-At + A - B)$

$y_p''(t) = -e^{-t}(-At + A - B) + e^{-t}(-A)$

$= e^{-t}(At - 2A + B)$

$\Rightarrow e^{-t}(At - 2A + B) - 3e^{-t}(-At + A - B) + 2e^{-t}(At + B) =$

$= e^{-t}(6At - 5A + 6B) = 2te^{-t}$

$\Rightarrow \begin{matrix} 6A = 2 \\ -5A + 6B = 0 \end{matrix} \Rightarrow \begin{matrix} A = \frac{1}{3} \\ B = \frac{5}{6}A = \frac{5}{18} \end{matrix}$

$\Rightarrow \boxed{y_p(t) = \left(\frac{1}{3}t + \frac{5}{18}\right)e^{-t}}$

Prob 2c) $y_p(t) = V_1 e^{2t} + V_2 e^{t}$

$$V_1' = \frac{-e^{t}(2te^{-t})}{\begin{vmatrix} e^{2t} & e^{t} \\ 2e^{2t} & e^{t} \end{vmatrix}} = \frac{-2t}{-e^{3t}} = 2te^{-3t}$$

$$\Rightarrow V_1 = 2\int \underset{\underset{1}{\downarrow}}{t}\, \overset{\overset{-\frac{1}{3}e^{-3t}}{\nearrow}}{e^{-3t}}\,dt = 2\left[-\frac{t}{3}e^{-3t} + \frac{1}{3}\int e^{-3t}\,dt\right]$$

$$= 2\left[-\frac{t}{3}e^{-3t} - \frac{1}{9}e^{-3t}\right]$$

$$= -\left(\frac{2}{3}t + \frac{2}{9}\right)e^{-3t}$$

$$V_2' = \frac{e^{2t}(2te^{-t})}{-e^{3t}} = -2te^{-2t}$$

$$\Rightarrow V_2 = -2\int \underset{\underset{1}{\downarrow}}{t}\, \overset{\overset{-\frac{1}{2}e^{-2t}}{\nearrow}}{e^{-2t}}\,dt = -2\left[-\frac{t}{2}e^{-2t} + \frac{1}{2}\int e^{-2t}\,dt\right]$$

$$= -2\left[-\frac{t}{2}e^{-2t} - \frac{1}{4}e^{-2t}\right]$$

$$= te^{-2t} + \frac{1}{2}e^{-2t} = e^{-2t}\left(t + \frac{1}{2}\right)$$

$$\Rightarrow y_p(t) = -\left(\frac{2}{3}t + \frac{2}{9}\right)e^{-3t}e^{2t} + \left(t + \frac{1}{2}\right)e^{-2t}e^{t}$$

$$= e^{-t}\left(\left(-\frac{2}{3} + 1\right)t + \left(\frac{1}{2} - \frac{2}{9}\right)\right)$$

$$\boxed{y_p(t) = e^{-t}\left(\frac{1}{3}t + \frac{5}{18}\right)}$$

d.) $y(t) = C_1 e^{2t} + C_2 e^{t} + e^{-t}\left(\frac{1}{3}t + \frac{5}{18}\right)$

Prob 2 cont

e.) $y_1 = y$

$y_2 = y' = y_1'$ $\Rightarrow$ $\boxed{\begin{aligned} y_1' &= y_2 \\ y_2' &= -2y_1 + 3y_2 \end{aligned}}$

$y_2' = y'' = 3y_2 - 2y_1$

f.) v-nullcline: $y_2 = 0$

h-nullcline: $y_2 = \frac{2}{3} y_1$

equilibrium: $(y_1, y_2) = (0,0)$

Problem 3

a.) $\ddot{x} + 144\pi^2 x = 11\cos(10\pi t)$

b.) $\ddot{x}_h + 144\pi^2 x_h = 0$

$r^2 + 144\pi^2 = 0 \Rightarrow r_{1,2} = \pm 12\pi i$

$\Rightarrow \boxed{x_h(t) = c_1\cos(12\pi t) + c_2\sin(12\pi t)}$

c.) $x_p(t) = A\cos(10\pi t) + B\sin(10\pi t)$

$x_p'(t) = -10\pi A\sin(10\pi t) + 10\pi B\cos(10\pi t)$

$x_p''(t) = -100\pi^2 A\cos(10\pi t) - 100\pi^2 B\sin(10\pi t)$

$$\Rightarrow \ddot{x}_p + 144\pi^2 x_p = -100\pi^2 A\cos(10\pi t) - 100\pi^2 B\sin(10\pi t) + 144\pi^2 A\cos(10\pi t) + 144\pi^2 B\sin(10\pi t) = 11\cos(10\pi t)$$

$\Rightarrow (-100\pi^2 + 144\pi^2)A = 11$
$(-100\pi^2 + 144\pi^2)B = 0$
$\Rightarrow A = \frac{11}{44\pi^2} = \frac{1}{4\pi^2}$
$B = 0$

$\Rightarrow \boxed{x_p(t) = \frac{1}{4\pi^2}\cos(10\pi t)}$

d.) $\Rightarrow x(t) = c_1\cos(12\pi t) + c_2\sin(12\pi t) + \frac{1}{4\pi^2}\cos(10\pi t)$

$x'(t) = -12\pi c_1 \sin 12\pi t + 12\pi c_2 \cos 12\pi t - \frac{10\pi}{4\pi^2}\sin(10\pi t)$

Init. cond. $\Rightarrow x(0) = 0 = c_1 + \frac{1}{4\pi^2} \Rightarrow c_1 = \frac{-1}{4\pi^2}$

$\dot{x}(0) = 0 = 12\pi c_2 \Rightarrow c_2 = 0$

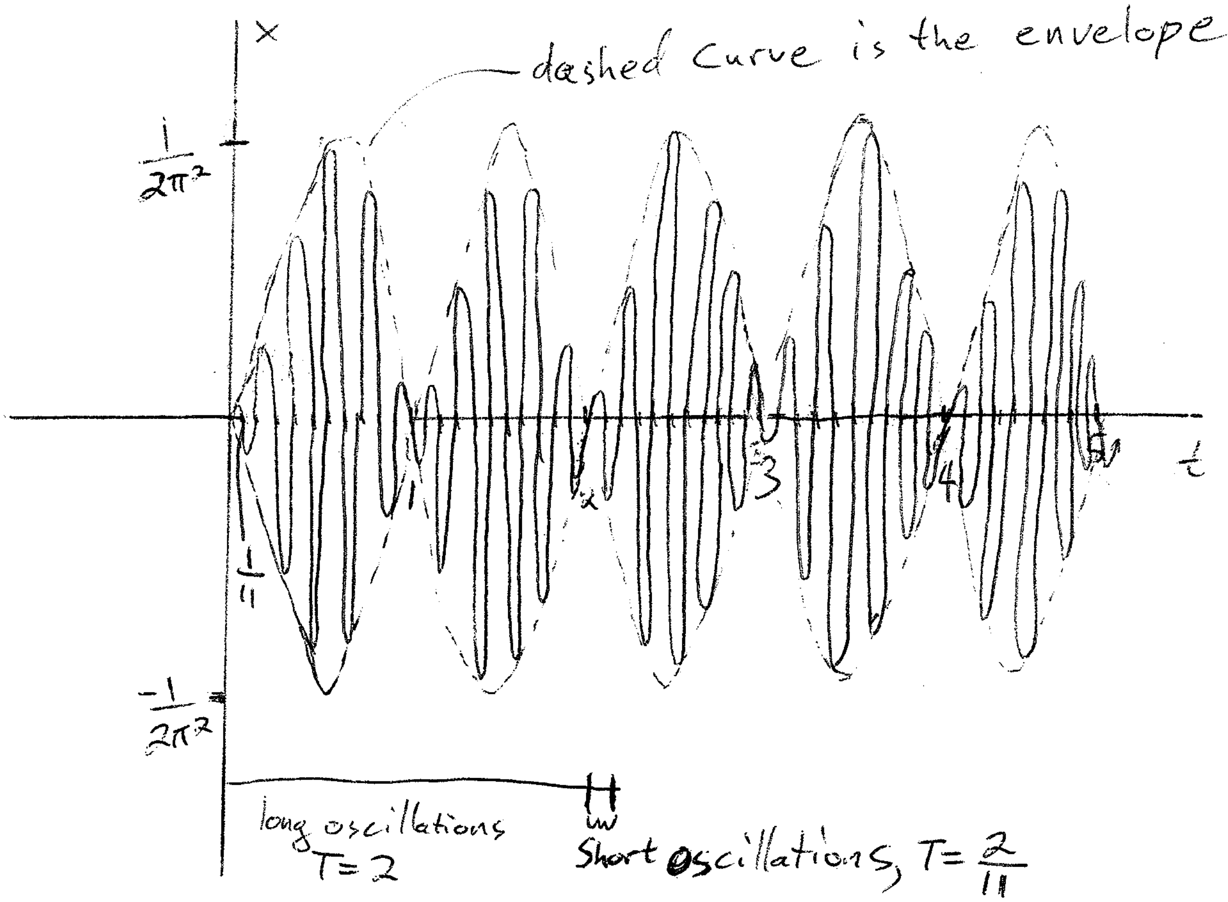
$\Rightarrow x(t) = \frac{1}{4\pi^2}\left(\cos(10\pi t) - \cos(12\pi t)\right)$

$\boxed{x(t) = \frac{1}{2\pi^2}\sin(\pi t)\sin(11\pi t)}$

using trig identity

3 cont

e) We call this beats



Problem 4

a.) $$x'' + \frac{1}{(x-2)^3} = 0 \quad , \quad x(0)=0 \, , \, x'(0) = \frac{1}{2}$$

$$v = x' \Rightarrow x'' = v' = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx} v$$

$$\Rightarrow x'' + \frac{1}{(x-2)^3} = v\frac{dv}{dx} + \frac{1}{(x-2)^3} = 0$$

$$\Rightarrow \boxed{v\frac{dv}{dx} = \frac{-1}{(x-2)^3}}$$

b.) $$\int v\,dv = \int \frac{-dx}{(x-2)^3} \Rightarrow \frac{v^2}{2} = \frac{(x-2)^{-2}}{2} + C$$

Using initial values $v(0) = \frac{1}{2}$ and $x(0) = 0$

$$\Rightarrow \frac{1}{8} = \frac{1}{8} + C \Rightarrow C = 0$$

$$\Rightarrow \boxed{v^2 = \frac{1}{(x-2)^2}} \Rightarrow \boxed{v = -\frac{1}{x-2}}$$

chose neg because of hint.

c.) $$v = x' = \frac{-1}{x-2} \Rightarrow (x-2)\,dx = -dt$$

$$\Rightarrow \frac{x^2}{2} - 2x = -t + C$$

Using IC's: $x(0) = 0 \Rightarrow C = 0$

$$\Rightarrow \boxed{\frac{x^2}{2} - 2x = -t}$$

d.) $$V(x) = \int \frac{1}{(x-2)^3}dx = \frac{(x-2)^{-2}}{-2} = \frac{-1}{2(x-2)^2}$$

e.) $$E(x,\dot{x}) = \frac{1}{2}\dot{x}^2 - \frac{1}{2(x-2)^2} \Rightarrow E_x = \underbrace{\frac{1}{(x-2)^3}}_{\text{Never equals zero}} \, , \, E_{\dot{x}} = \dot{x}$$

so $\boxed{\text{NO Equilibrium}}$