APPM 2360 Final Summer 2009

ON A SEPARATE COVER SHEET WRITE:
(1) your name,
(2) your student ID number,
(3) your lecture section,
(4) your instructor's name and
(5) a grading table.
You have till Friday, at the start of Lecture to work both problems below. The point values are indicated at the start of each problem; there are 80 points total. Show ALL of your work and box in final answers. Start each problem on a new page. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. These problems are open book, open notes but you may not consult anyone else for help. **STAPLE your final write-up.**

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1. (40 points) Consider the following 2nd-order, linear, non-homogeneous differential equation with variable coefficients.

$$ty'' + y' + \left(t - \frac{1}{4t}\right) y = \sqrt{t}$$

The functions

$$y_1(t) = t^{-1/2} \sin t \qquad \text{and } y_2(t) = t^{-1/2} \cos t$$

are a fundamental set of solutions to the corresponding homogeneous equation.

(a) Verify that the given functions y_1 and y_2 are indeed solutions to the homogeneous equation.

(b) Verify that the given functions y_1 and y_2 are linearly independent.

(c) Write down the solution $y_h(t)$ to the homogeneous equation.

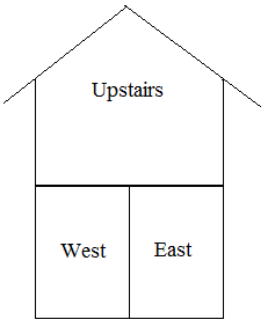
(d) Now, going back to the non-homogeneous differential equation, write down the system of equations that we solve for Variation of Parameters. Show how Cramer's Rule gives a solution to this system for v_1' and v_2'.

(e) Integrate the results of part (d) to find the unknown functions v_1, v_2.

(f) Write down the particular solution $y_p(t)$ to the non-homogeneous equation. Simplify your answer.

(g) Write down the general solution to the original 2nd-order, non-homogeneous differential equation.

2. (40 points) Professor Hammond lives in a three-room house shown in the figure below. One winter night, with the outside temperature 0°F, his furnace fails. Suppose that the time constants in Professor Hammond's house, which specify the rate of heat flow between the rooms, are as given in the table below. Suppose that the temperatures in the Upstairs, West and East rooms were 60°F, 70°F and 80°F, respectively at the time when the furnace broke down.



The heat	flow rates	given in	hours	
room	Outside	Upstairs	West	East
Outside		1	1	1
Upstairs	1		$1/2$	$1/2$
West	1	$1/2$		$1/3$
East	1	$1/2$	$1/3$	

(a) Using Newton's Law of Cooling, write the initial value problem which models the temperatures in each room. Use $U(t)$, $W(t)$ and $E(t)$ to denote the Upstairs, West and East temperatures at time t. (Note: you may find p. 366 in the text very useful at this point in your life...)

(b) Write the system in matrix-vector form and find the eigenvalues and eigenvectors of the matrix.

(c) Find the solution to the IVP. Write $U(t)$, $W(t)$ and $E(t)$ separately in your final answer.

(d) What is the temperature in each room after 1 hour? Is the insulation good or bad?