APPM 2450 Calculus 3 Computer Lab
Lab Exercise 6

Create a Mathematica notebook that does all of the following. Feel free to ask your neighbor or your lab instructor for help if you get stuck. Items with a ➠ are required, items with a ☆ are optional.

☆ Define the function $f(x,y) = 2 + 2x + 2y - x^2 - y^2 + \sin(0.99\pi x - 4) + 15$. We will consider this function on the rectangular domain $0 \le x \le 5$, $-3 \le y \le 3$.

☆ Make a nice 3D plot of f on the domain. Remember you can use `RealTime3D‘` to rotate your graph. Type `Default3D‘` to return to the normal graphing style with axes labels and all.

☆ Make a nice contour plot of f on the domain,

☆ Find all interior points where $\nabla f = 0$. There are two. You will have to use `FindRoot[]`, which means you need an initial guess. You can either (i) use the contour plot to find approximate values of the critical points, or (ii) do something like the following

```
<<Graphics‘ImplicitPlot‘
p1=ImplicitPlot[D[f[x,y],x]==0,{x,0,5},{y,-3,3},PlotStyle->Hue[1]]
p2=ImplicitPlot[D[f[x,y],y]==0,{x,0,5},{y,-3,3},PlotStyle->Hue[0.4872]]
Show[p1,p2,AxesLabel->{"x","y"}]
```

If you do this, you have critical points where the red and blue lines intersect.

☆ Use the second derivative test to determine if the critical points are local maxima, local minima, or saddles (you should find one is a max, the other is a saddle).

☆ Plot $f(x,y)$ near these critical points. For example, if you found a critical point at (1,2), you could do something like

```
Plot3D[f[x,y],{x,0.9,1.1},{y,1.9,2.1},AxesLabel->{"x","y","f(x,y)"}]
```

☆ Consider the boundary where $x = 5$. On this boundary we have $g(x) = f(5,y)$. This is a function of only one variable. Use what you know from Calc I to find any critical points along the boundary.

☆ Plot $f(x,y)$ near these critical points; be careful to not plot any part of f that is outside of the domain! For example, if you found a critical point at $y = 2.1783$, this would mean the point you are interested in is (5,2.1783). You could do something like

```
Plot3D[f[x,y],{x,4.8,5},{y,2.0783,2.2783},AxesLabel->{"x","y","f(x,y)"}]
```

☆ Remember the definition of a local max/min. Look at your plot carefully. Is the point(s) you found really a local max/min?

☆ Yup, you guessed it, now you have to do the above 3 steps for the other three boundaries!

☆ Once you have found all interior local extrema, and all possible extrema on the boundaries, evaluate f at all of these points and pick out the global max and min.

☆ If (and only if) you complete all of the above, congratulations! Go to ‘Kernel’, then ‘Delete All Output’ before you save. Save your notebook as *YourLastName_Rocks.nb* and email as an attachment to your instructor; he will be quite impressed, and you will be ready to tackle Lab 2.