APPM 2450 Calculus 3 Computer Lab
Lab Exercise 3

Create a Mathematica notebook that does all of the following. Feel free to ask your neighbor or your lab instructor for help if you get stuck. Items with a ➠ are required, items with a ✰ are optional.

- ➠ Consider a particle moving in the plane whose path is given by the vector function $\mathbf{r}(t) = \cos^3(t)\mathbf{i} + \sin^3(3t)\mathbf{j}$. Enter this into Mathematica in an appropriate way (as a vector - curly brackets, and as a function of t - `r[t_]`). Note: $\cos^3(t)$ would be entered as `Cos[t]^3`.
- ➠ Use `ParametricPlot` to plot this curve for $0 \leq t \leq 2\pi$. You may want to store this plot for future use, `p1=ParametricPlot[...]`.
- ➠ On the previous plot, show the position of the particle when $t = 0$ and when $t = \pi/6$. See today's example notebook for some help.
- ➠ Define the function `v[t]` as the velocity of the particle, you can do this with a command something like `v[t_]=D[r[t],t]`.
- ➠ As above, define the acceleration of the particle.
- ➠ Remember from class that speed is the magnitude, or length, of velocity, speed=$|\mathbf{v}|$. Make a plot (this is a plot, not a parametric plot) of the particle's speed as a function of time.
- ➠ Remember from class that arclength=$\int_a^b |\mathbf{v}|\, dt$. Use this to calculate the total distance traveled (arclength) of the particle for $0 \leq t \leq 2\pi$. HINT: Since the integrand is 'messy', you will want to use `\NIntegrate` instead of `\Integrate` here.
- ➠ Now consider a particle moving in space whose path is given by $\mathbf{r}(t) = \cos^3(t)\mathbf{i} + \sin^3(3t)\mathbf{j} + (\sin(t) - \cos(t))\mathbf{k}$. Use `ParametricPlot3D[]` to plot the particle's path.
- ✰ Mark on the above plot the position of the particle when $t = 0$, $t = \pi/6$ and $t = \pi/2$. Before trying this, you will need to enter the command ``<< Graphics`Graphics3D` `` (those are left quotes, found on the key next to the 1 on your keyboard).
- ➠ Plot the speed of the particle as a function of time. Is the speed of the particle ever zero?
- ✰ Find the arclength of the curve traced out by the particle for $0 \leq t \leq 2\pi$.
- ✰ Remember that the curvature, $\kappa(t)$, can be calculated by $\kappa(t) = \dfrac{|\mathbf{v}(t) \times \mathbf{a}(t)|}{|\mathbf{v}(t)|^3}$. Use this to find the curvature as a function of time.
- ✰ Plot the curvature as a function of time.
- ✰ ✰ Consider the circle, $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$. What is the curvature of this circle? Remember that to do a cross product, your vector must be of length 3. In other words, you will need to explicitly tell Mathematica that the z-component of your vector is zero.
- ➠ Go to 'Kernel', then 'Delete All Output'. Save your notebook as *YourLastName3.nb* and email as an attachment to your instructor.