

APPM 2450 Calculus 3 Computer Lab
Lab Exercise 6

Create a Mathematica notebook that does all of the following. Feel free to ask your neighbor or your lab instructor for help if you get stuck. Items with a \blacktriangleright are required, items with a \star are optional.

\Rightarrow This worksheet is intentionally long and hard. This week, it is totally optional. Do it if you have the time/inspiration/courage. Completing it will help greatly with Lab 2, and if you can't do this, you will have a hard time with the material and computations in Lab 2.

\star Define the function $f(x, y) = 2 + 2x + 2y - x^2 - y^2 + \sin(0.99\pi x - 4) + 15$. We will consider this function on the rectangular domain $0 \leq x \leq 5$, $-3 \leq y \leq 3$.

\star Make a nice contour plot of f on the domain,

\star Find all interior points where $\nabla f = 0$. There are two. You will have to use `FindRoot[]`, which means you need an initial guess. You can either (i) use the contour plot to find approximate values of the critical points, or (ii) do something like the following

```
p1=ContourPlot[D[f[x,y],x]==0,{x,0,5},{y,-3,3}]
p2=ContourPlot[D[f[x,y],y]==0,{x,0,5},{y,-3,3}]
Show[p1,p2,AxesLabel->{"x","y"}]
```

If you do this, you have critical points where the lines intersect.

\star Use the second derivative test to determine if the critical points are local maxima, local minima, or saddles (you should find one is a max, the other is a saddle).

\star Plot $f(x, y)$ near these critical points. For example, if you found a critical point at (1,2), you could do something like

```
Plot3D[f[x,y],{x,0.9,1.1},{y,1.9,2.1},AxesLabel->{"x","y","f(x,y)"}]
```

\star Consider the boundary where $x = 5$. On this boundary we have $g(x) = f(5, y)$. This is a function of only one variable. Use what you know from Calc I to find any critical points along the boundary.

\star Plot $f(x, y)$ near these critical points; be careful to not plot any part of f that is outside of the domain! For example, if you found a critical point at $y = 2.1783$, this would mean the point you are interested in is (5,2.1783). You could do something like

```
Plot3D[f[x,y],{x,4.8,5},{y,2.0783,2.2783},AxesLabel->{"x","y","f(x,y)"}]
```

\star Remember the definition of a local max/min. Look at your plot carefully. Is the point(s) you found really a local max/min?

\star Yup, you guessed it, now you have to do the above 3 steps for the other three boundaries!

\star Once you have found all interior local extrema, and all possible extrema on the boundaries, evaluate f at all of these points and pick out the global max and min.

\star If (and only if) you complete all of the above, congratulations! Go to 'Kernel', then 'Delete All Output' before you save. Save your notebook as *LastName_FirstName_worksheet6.nb* and email as an attachment to your instructor; he will be quite impressed, and you will be ready to tackle Lab 2.