DETECTING ART FORGERIES

Christie's has scheduled an old master paintings auction later this year, but some scholars questions the authenticity of some of the paintings. Among these paintings are three Vermeers and seven Delacroixs. Using first order differential equations, you will determine if any of the paintings are forgeries before the auction catalog is announced on **February 20, 2006**.

1. Instructions

This lab is due at the beginning of lecture on *Monday, February 20, 2006.* You *must* also turn in an electronic copy to AMESS. Lab hours will be held in ECCR 143 from Monday, February 13, 2006 to Sunday, February 19, 2006.

You are strongly encouraged to work in groups of no more than three (3) people. If you work in a group, you need only hand in one lab per group. All labs must include a title page which includes *all* of the following information: *Name, Student I.D. Number, Professor, Recitation Number, and TA's Name.*

Remember, all your calculations (including those in your appendix) must be on printer paper. You are also encouraged to review the writing guidelines, as up to 20% of your grade may be based on organization, structure, style, grammar and spelling.

2. Delacroix and Vermeer

Eugéne Delacroix was the greatest French romantic painter. His use of color influenced the development of both impressionism and post-impressionism. The inspiration for his paintings (most between the 1820s and 1860s) came from historical or contemporary events, literature and a visit to Morocco in 1832.

Jan Vermeer was a Dutch painter who lived and worked (during the 1650s and 1660s) in Delft and created some of the most exquisite paintings in Western art. His works are rare; of the 35 or 36 paintings attributed to him, most portray figures in interiors and are admired for the sensitivity with which he rendered the effects of light and color.

3. History of Forged Vermeers

After Belgium was liberated during World War II, the Dutch Field Security began searching for Nazi collaborators. It was discovered that a third rate Dutch painter H. A. Van Meegeren sold the painting "Woman Taken in Adultery" by Vermeer to the Germans. On May 29, 1945 Van Meegeren was arrested for collaborating with the enemy. But, Van Meegeren shocked the world on July 12, 1945 by announcing that he had forged "Woman Taken in Adultery" as well as other Vermeer and de Hooghs including the very famous "Disciples at Emmaus."

Since no one believed Van Meegeren, he began forging "Jesus Amongst the Doctors" while in prison to escape the charge of treason. After they changed the charge from collaboration to forgery, Van Meegeren refused to finish and age the painting to prevent the investigators from discovering his secrets to aging his forgeries. After several months, an international panel of chemists, physicists and art historians unanimously concluded that the paintings were forgeries.

The panel found that two of the paintings contained the color cobalt blue which was not known in the 17th century. They also found that while the paintings resisted water and ethyl alcohol (just like 17th century ones) they had not been attacked by strong acids and alkalis like a 17th century painting would have been. On the basis of this evidence Van Meegeren was convicted on October

Source	Date	Concentration (dpm/g of Pb) $^{210}_{84}$Po	$^{226}_{88}$Ra	$^{210}_{84}$Po at date of manufacture	Separation Factor
Lead Metal (US)	1966	6.8 ± 1.2	1.2 ± 0.6	6.8 ± 1.2	5.7
Stack Process (Australia)	1948	5.2 ± 0.5	0.1 ± 0.02	8.6 ± 0.9	86
Portrait by Resco (US)	1923	9.2 ± 1.2	0.36 ± 0.40	30 ± 5	83
Flowers by Speicher (US)	1920	3.3 ± 0.6	0.12 ± 0.5	13 ± 3	110
Landscape by J. Sloane (US)	1910	7.3 ± 0.8	0.13 ± 0.07	40 ± 4	310
Landscape (US)	1850-60	8.7 ± 1.6	2.2 ± 0.5	210 ± 80	95
Landscape (England)	1850-60	2.1 ± 0.3	0.62 ± 0.15	48 ± 11	77
Primitive (France)	1830	0.51 ± 0.10	0.03 ± 0.02	32 ± 8	1000
Eicholz (US)	1817	0.35 ± 0.08	0.13 ± 0.03	22 ± 8	170
Portrait (France)	1780-88	2.4 ± 0.6	1.3 ± 0.4		
Portrait of Claypoole (US)	1746	1.96 ± 0.46	1.82 ± 0.43		
Portrait of Badger (US)	1730-50	2.58 ± 0.81	2.70 ± 0.29		
Portrait (Italy)	1600	0.21 ± 0.10	0.21 ± 0.29		

TABLE 1. Artists White Lead (18th to 20th Century) [1].

12, 1947 and sentenced to one year in prison. While in prison he suffered a heart attack and died on December 30, 1947.

Regardless of the evidence, the famed "Disciples at Emmaus" was certified as an authentic Vermeer by the noted art historian A. Bredius and was bought by the Rembrandt Society for $170,000. It was not until 1968 that "Disciples at Emmaus" was proven to be a forgery by measuring the paints natural radiation [2].

4. Model

The physicist Rutherford and his colleagues showed that some elements are unstable and spontaneously disintegrate to form atoms of a new element. This decay is directly proportional to the number of atoms of the substance. If we let $N(t)$ denote the number of atoms present at time t, then $\lambda N(t)$ is the number of atoms that disintegrate per minute (dpm) to a new element. The first order differential equation which models $N(t)$ is then

$$N'(t) = -\lambda N(t), \tag{1}$$

where λ is the positive decay constant of the substance. The radioactive substance white lead ($^{210}_{82}$Pb) has a half-life of 22 years and is a pigment of great importance in painting. (In practice, the alpha emitting polonium-210 $\{^{210}_{84}\text{Po}\}$ is measured instead of the beta emitting lead-210 since after very few years their disintegrations per minute are virtually equivalent.)

Lead is manufactured from ores which contain uranium and its descendents (the elements to which uranium decays). In particular, the descendent radium-226 ($^{226}_{88}$Ra) decays to lead-210 with a half-life of 1600 years. While in the ore, the lead is in radioactive equilibrium with the radium; that is, the amount of radium decaying to lead per unit time is equal to the amount of lead disintegrating per unit time. However, most of the radium and its descendants are removed when extracting the lead for the paint (see the separation factor in Table 1); at which point the lead-210 is no longer supported by the radium-226 and begins decaying with a half-life of 22 years.

Painting	Concentration (dpm/g of Pb) $^{210}_{84}$Po	$^{226}_{88}$Ra
Vermeers:		
Lace Maker	1.5 ± 0.3	1.4 ± 0.2
Laughing Girl	5.2 ± 0.8	6.0 ± 0.9
Woman Reading Music	1.9 ± 0.2	0.88 ± 0.09
Delacroixs:		
The Death of Sardanapal	6.6 ± 0.8	4.4 ± 0.4
Ovid Among the Scythians	8.7 ± 0.8	4.0 ± 0.4
Arabian Horses Fighting in a Stable	61 ± 2	39 ± 2
Orphan Girl in a Cemetery	3.0 ± 0.2	1.8 ± 0.1
The Massacre at Chios	0.54 ± 0.05	0.05 ± 0.01
The Abduction of Rebecca	0.50 ± 0.05	0.33 ± 0.03
Combat of Giaour and Hassan	4.6 ± 0.4	1.4 ± 0.2

TABLE 2. Paintings up for Auction. (Note: All measurements were taken in 2006.)

5. QUESTIONS

1.) Given the initial condition, $N(t_0) = N_0$, find the analytic solution of (1).
2.) Using the half-life of lead-210, determine λ. [Remember, there are 525600 minutes in a year.] What are the units of λ? [Hint: The units of λ are *not* min^{-1}.]
3.) Let $y(t)$ denote the amount of lead-210 (in atoms) per gram of ordinary lead (Pb) at time t. Then

$$y'(t) = -\lambda y(t) + r(t), \qquad y(t_0) = y_0, \tag{2}$$

where $y(t_0) = y_0$ is the amount of lead-210 (in atoms) per gram of ordinary lead at the time of manufacture/painting and $r(t)$ is the number of disintegrations of radium-226 per minute per gram of ordinary lead (dpm/g of Pb) at time t.

Since the half-life of radium-226 is 1600 years, is it reasonable to assume that $r(t)$ is constant over the time period (150-350 years) that we are interested in? Assume that $r(t) = \bar{r}$ and find the solution to (2), where $\bar{r}$ is the measured dpm/g of Pb.
4.) We can easily measure the number of disintegrations of polonium-210 and radium-226 per minute per gram of ordinary lead (dpm/g of Pb). Is this enough information to determine the age of the painting? If not, what additional information do we need?
5.) If we know the amount of radium-226 (in dpm/g of Pb) in the original ore, can we determine the age of the painting? [Hint: Is y_0 related to the amount of radium-226 in the original ore?] If so, what is the formula for the age of the painting?
6.) Let us define the *separation factor* (SF) by the ratio of polonium-210 *at the time of manufacture* to radium-226, where both are measured in dpm/g of Pb. So, the SF of the portrait by Resco in Table 1 is (30 dpm/g of Pb)/(0.36 dpm/g of Pb) = 83. Likewise, if we assume that the SF of "Lace Maker" and "The Death of Sardanapla" is 100, then the concentration of lead-210 when they were painted is 140 and 440 dpm/g of Pb respectively. Based on this assumption, what is the age of "Lace Maker" and "The Death of Sardanapla?"

Is it reasonable to assume that the SF is 100? Assuming the SF is 100, compute the age a couple paintings from the 18th, 19th and 20th centuries. How well do your results compare to the actual age of the painting? [Note: The data in Table 1 is from 1966.]

7.) Can you determine which (if any) of the paintings in Table 2 are forgeries? If so, make a table showing which paintings were 20th century forgeries and which were painted in the 18th and 19th century.

8.) Will it be possible to distinguish a 18th century painting from a 19th century painting in 100 years using these methods? In (2), assume that

$$r'(t) = -\mu r(t), \tag{3}$$

where μ is the positive decay constant of radium-226. [Note: $r(t)$ is in dpm/g of Pb, while $y(t)$ is in atoms.] Using the half-lives of lead-210 and radium-226, solve for $y(t)$ and $r(t)$. Using these solutions, determine what the dpm/g of Pb of $^{210}_{84}$Po and $^{226}_{88}$Ra will be in 100 years for all the paintings in Table 2.

9.) (Extra Credit) Using your solution for $r(t)$ from question 8, calculate the dpm of $^{226}_{88}$Ra per gram of Pb for "Lace Maker" and "The Death of Sardanapla" when they were painted (based on your answers in question 6).

If there is any uranium remaining in the paint, is your calculation for the amount of radium-226 too high or two low? If we also measure the amount of uranium, $\bar{u}$, (which has a half-life of 4.51×10^9 years) in dpm/g of Pb, what would be the system of first order differential equations which would model $y(t)$ and $r(t)$? [For Extra-Extra Credit, solve this system given that $y(t_0) = y_0$ and $r(t_0) = r_0$.]

References

[1] Keisch, B., Feller, R. L., Levine, A. S., Edwards, P. S., Dating and Authenticating Works of Art by Measurement of Natural Alpha Emitters, Science (155), 1238–1241, March 1967.

[2] Keisch, B., Dating Works of Art Through Their Natural Radioactivity: Improvements and Applications, Science (160), 413–415, April 1968.