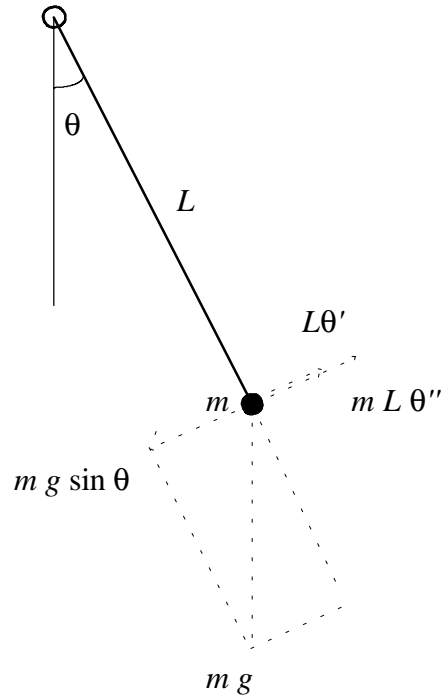


DERIVATION OF THE EQUATION FOR A SIMPLE PENDULUM

The simple pendulum consists of a bob of mass m hanging on a presumed massless rigid rod of fixed length L . The rod can turn freely (within the plane of the figure below) around some peg at its top end. It depends on the problem if it is allowed to make full circular swings around the peg, or if it is constrained to a restricted range of θ (for example $-\pi \leq \theta \leq \pi$ if there is a ceiling just above the peg).



The forces that act at the bob, in the direction orthogonal to the rod, are

inertia	$m L \theta''$
gravity	$m g \sin \theta$
resistance	$b L \theta'$

(where we have assumed that the latter is proportional to the bob's velocity).

The forces should add up to zero. Hence we get the governing ODE:

$$m L \theta'' + b L \theta' + m g \sin \theta = 0 \tag{1}$$

It remains to verify that all the signs (dependent on the directions of the forces) are correct.

Let's denote the direction of increasing θ as "+", decreasing θ as "-". For example, when the pendulum is moving out to the right, we get for the three terms:

Term	Direction of actual force	Sign of term
$m L \theta''$	+	-
$b L \theta'$	-	+
$m g \sin \theta$	-	+

All three terms therefore express forces in the "-" direction. So they can be added up without making any sign adjustments. The same holds for the other parts of the motion; equation (1) is valid in all cases.