APPM 3010 - Review for Exam 1

1. Fundamentals
   - Given an ODE, possibly a higher order scalar equation and/or a nonautonomous equation, be able to put it in the form of a first order, autonomous system.
   - Given a multi-step map, be able to write it as a system of 1-step maps.
   - Be able to provide examples of various types of dynamical systems (e.g. maps and ODES, linear and nonlinear, autonomous and nonautonomous, high order scalar equations and systems of equations).
2. Well posedness
   - Be able to state the existence and uniqueness theorem for ODEs.
   - Be able to state the theorems given in class regarding continuity in initial conditions and parameters.
   - Be able to explain why these ideas are important in modeling physical systems.
   - Know the difference between local and global existence.
   - Given an explicit $\mathbf{f}(\mathbf{x}, t, \lambda)$, be able to determine a range of initial conditions where the initial value problem (IVP) $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t, \lambda)$ is locally well posed. Be able to determine a range of initial conditions where the corresponding solutions exist for all time.
   - Be able to give examples of IVPs where uniqueness fails.
   - Be able to give examples of IVPs whose solutions blow up in finite time.
3. Flows on the real line
   - Given $\dot{x} = f(x)$ either explicitly or as a graph, be able to determine the equilibria of the flow and their stability by graphing $f(x)$.

- Be able to determine the curvature of solutions by computing $\ddot{x} = f'(x)\dot{x}$.
- Given a phase portrait, be able to give an example of an IVP $\dot{x} = f(x)$ that is consistent with that phase portrait.
- Give a collection of solutions $x(t)$, be able to give an example of an IVP $\dot{x} = f(x)$ that is consistent with those solutions.
- Be able to determine the stability of an equilibrium point by linearization. From linearization, be able to determine the asymptotic rate of decay to or growth from the equilibrium point.

4. Bifurcations

- Given a first order equation with a parameter, be able to determine when, if at all, a bifurcation occurs.
- Know the basic characteristics of the saddle-node, transcritical, and pitchfork bifurcations, including their bifurcation diagrams.
- Be able to determine if one of these bifurcations occurs in a given system that is not in the canonical form.