APPM 3010 - Review for Exam 2

1. One dimensional maps

    - Be able to find fixed points of simple maps explicitly and determine their stability using linear information.
    - Be able to identify fixed points of more complicated maps graphically and determine their stability.
    - Know the equation satisfied by a period - m orbit. Given the graph of the mth iteration map, be able to find period - m solutions. Know the stability criterion for period - m orbits and be able to apply it.
    - Be able to describe in words the period doubling route to chaos.
    - Be able to describe how the Lyapunov exponent can indicate a system's tendency towards chaos. **You need not memorize the definition of the Lyapunov exponent - it will be given if necessary.**
    - Given the graph of a map's Lyapunov exponent versus a parameter, be able to indicate where the onset of chaos is expected and where intermittancy is expected.

2. Planar linear systems and linearization of planar nonlinear systems

    - Be able to reproduce Figure 5.2.8 on page 137 of Strogatz. Understand how the eigenvalues and eigenvectors of a matrix determine the local phase portrait of the corresponding dynamical system.
    - Know the difference between Lyapunov stability and asymptotic stability. Be able to provide an example of an equilibrium point that is Lyapunov stable but not asymptotically stable, and vice versa.
    - Know what attracting means and be able to provide examples. Be able to provide an example of a fixed point that is attracting but not Lyapunov stable. Know that Lyapunov stable and attracting together imply asymptotically stable.

- Know when the information from the linearization will predict the correct stability type of an equilibrium point under small nonlinear perturbations.
- Know when the information from the linearization will predict the correct structure type of an equilibrium point under small nonlinear perturbations.
- Be able use information from the linearization to draw a rough picture of the local phase portrait for the corresponding nonlinear system.

3. Periodic orbits in planar systems

   - Know the two types of periodic solutions in a planar nonlinear system. Be able to give an example of each.
   - Be able to state the Poincare-Bendixson theorem and know how to apply it. Know the implications of this theorem in terms of chaotic behavior.
   - Given a nearly decoupled system in polar coordinates, be able to design a trapping region for the corresponding flow.
   - For more complicated flows, be able to show that a given region is a trapping region.
   - Know what nullclines are and how they are useful in determining the global flow.
   - Know the hypotheses and conclusion of the Hopf bifurcation theorem.
   - Know how to set up a Poincare section for a flow. Know how to use the Poincare map to determine if periodic orbits exist and when they do, their stability type.