ANSWER KEY

APPM 3170: Discrete Applied Mathematics - Fall 2008

Exam #3
Lecturer: Manuel Lladser

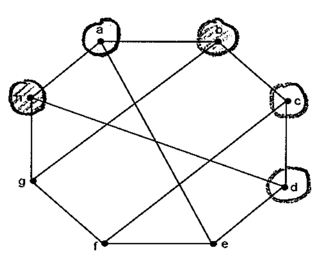
INSTRUCTIONS: On the front of your bluebook please print your *name*, *student ID*, *course code*, *exam number*, *date* and *lecturer's name.* Please draw a grading table (with *2 columns* and *6 rows*). Show all your work in your bluebook. Please start each new problem in a *new page.* Solve the problems in the *same order* as they are requested. A correct answer with no supporting work may receive no credit while and incorrect answer with some correct work may receive partial credit. *Textbooks, class notes, graphing or programmable calculators, and crib sheets are not permitted.*

1. (10 points.) **(a)** What is the largest prime factor that a composite number between 2 and 120 can have? Explain. **(b)** Apply the *Sieve Method* on the table below to determine the integers between 2 and 120 that are primes.

	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~
101	~~102~~	103	~~104~~	~~105~~	~~106~~	107	~~108~~	109	~~110~~
~~111~~	~~112~~	113	~~114~~	~~115~~	~~116~~	~~117~~	~~118~~	~~119~~	~~120~~

2. (20 points.) How many integers between 1 and 1,000,000 are <u>divisible</u> by 2, 6, or 9?
Simplify your answer to a combination of terms of the form $\lfloor m/n \rfloor$ but <u>do not</u> compute explicitly.

3. (20 points.) Is the following graph bipartite? Justify! i.e. exhibit an appropriate partition of the vertices or provide a rigorous argument that the graph is not bipartite.



not possible !!

(FOUR MORE PROBLEMS ON THE BACK!)

P1 (a) = 59 b/c 59 is prime and $2 \times 59 < 120 > 2 \times 61$

(b) $= \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113\}$.

P2

$$\text{ANS} = \left\lfloor \frac{10^6}{2} \right\rfloor + \cancel{\left\lfloor \frac{10^6}{6} \right\rfloor} + \left\lfloor \frac{10^6}{9} \right\rfloor - \cancel{\left\lfloor \frac{10^6}{6} \right\rfloor} - \left\lfloor \frac{10^6}{18} \right\rfloor - \cancel{\left\lfloor \frac{10^6}{18} \right\rfloor} + \cancel{\left\lfloor \frac{10^6}{18} \right\rfloor}$$

$$= \left\lfloor \frac{10^6}{2} \right\rfloor + \left\lfloor \frac{10^6}{9} \right\rfloor - \left\lfloor \frac{10^6}{18} \right\rfloor.$$

P3 NO! b/c if $a \in V_1$ then $b, h \in V_2$. But $b \in V_2$ implies $c \in V_1$ and $h \in V_2$ implies $d \in V_1$ $\ast$ b/c d and c are connected through an edge.

P4 # vertices $= m + n$

edges $= m \cdot n$

P5 (a) YES! b/c each vertex has even degree. The degree sequence is: 4, 4, 6, 4, 2, 4, 4, 6, 4

(b) YES! due to part (a)

(c) No! e.g. $\deg(1) + \deg(5) = 4 + 2 = 6 <$ # vertices and 1 and 5 are not connected through an edge

(d) YES! e.g. 9, 8, 6, 5, 7, 4, 3, 2, 1, 9 is a Hamilton circuit.

P6 Let $G = (V, E)$ be a bipartite graph with $|V| = \text{odd}$. In particular, $V = V_1 \cup V_2$ with $V_1 \cap V_2 = \emptyset$ and no two vertices in V_i are neighbors. Now suppose that $v_1, \dots, v_n, v_{n+1} = v_1$ is a Hamilton circuit. WLOG assume $v_1 \in V_1$. Then $v_2 \in V_2$. In general, $v_k \in V_1$ when k is odd. and $v_k \in V_2$ when k is even. Hence $(n+1)$ must be odd i.e. n must be even $\ast$ Hence a Hamilton circuit cannot exist.

P7

Yes!! Notice that 1, 2, 3, 4, 5, 1 and 6, 8, 10, 7, 9, 6 are disjoint cycles in the RHS. On the other hand, 1, 2, 8, 5, 6, 1 and 7, 10, 9, 4, 3, 7 are also disjoint cycles on the LHS. This motivates to define

$f(1) = 1$, $f(2) = 2$, $f(3) = 8$, $f(4) = 5$, $f(5) = 6$

$f(6) = 7$, $f(7) = 4$, $f(8) = 10$, $f(9) = 3$, $f(10) = 9$

This transformation preserves all the edges in the cycles. To show it is an isomorphism, it is enough to show that it preserves the edges

RHS	f(RHS)	Edge on the LHS ?
$\{1, 10\}$	$\{1, 9\}$	✓
$\{2, 9\}$	$\{2, 3\}$	✓
$\{3, 8\}$	$\{8, 10\}$	✓
$\{4, 7\}$	$\{5, 4\}$	✓
$\{5, 6\}$	$\{6, 7\}$	✓

So f is an isomorphism.