APPM 3170: Discrete Applied Mathematics - Fall 2008

Quiz #2
Lecturer: Manuel Lladser

YOUR NAME: ANSWERKEY

(a) Prove that if $x, y \in \mathbb{R}$ then $\max\{x, y\} + \min\{x, y\} = x + y$.

If $x \leq y$ then $\max\{x,y\} = y$ and $\min\{x,y\} = x$. Hence $\max + \min = y + x = x + y$. By symmetry, the same holds when $x \geq y$.

(b) Let n be an integer. Prove by contraposition that if $n^3 + 5$ is odd then n is even.

Assume n is odd i.e. 2 is not a factor of n. Then 2 cannot be a factor of n^3 either. Hence n^3 is odd. Since the sum of two odd numbers is even, we conclude that (n^3+5) is even

(c) TRUE or FALSE: $\{5, 1, 3, 3, 3, 3, 1, 1, 5, 5, 5\} = \{1, 3, 5\}$?

TRUE

(d) What can you conclude about the sets A and B if it is known that $(A - B) = (B - A)$?

A and B must be equal

(e) Show that if $f : A \to B$ and $g : B \to C$ are onto then $(g \circ f)$ is also onto.

By definition $(g \circ f): A \to C$ and $(g \circ f)(x) = g(f(x))$. Let $z \in C$. Since g is onto, $\exists\, y \in B$ such that $g(y) = z$. Since f is onto, $\exists\, x \in A$ such that $f(x) = y$. Hence $g(f(x)) = g(y) = z$. This show that $\exists\, x \in A$ such that $(g \circ f)(x) = z$. Hence, $(g \circ f)$ is onto.