APPM 3170: Discrete Applied Mathematics - Fall 2008

Quiz #3
Lecturer: Manuel Lladser

YOUR NAME: ANSWER KEY

(a) Use the Euclidean algorithm to find $gcd(1529, 14039)$.

$14039 = 1529 \cdot 9 + 278$
$1529 = 278 \cdot 5 + 139$
$278 = 139 \cdot 2 + 0$

hence the $\gcd(1529, 14039) = 139$

(b) Show that for all integer $n \geq 1$, $\sum_{i=1}^{n} i \cdot (i!) = (n+1)! - 1$.

Basic step: $\sum_{i=1}^{1} i(i!) = 1(1!) = 1 = 2! - 1$ ✓

Inductive step: $\sum_{i=1}^{m+1} i(i!) = \sum_{i=1}^{m} i(i!) + (m+1)((m+1)!)$

$\overset{\text{induction}}{=} (m+1)! - 1 + (m+1)((m+1)!) = (1+m+1) \cdot (m+1)! - 1$
$= (m+2) \cdot (m+1)! - 1$
$= (m+2)! - 1$ ✓

(c) Use the Merge Sort algorithm to sort the list of numbers 8, 0, 4, 12, 7, 2, 10, 3. Show all the steps used by the algorithm. HINT: Consider using a tree to represent the operations executed by Merge Sort.