APPM 3170: Discrete Applied Mathematics - Fall 2008

Quiz #4
Lecturer: Manuel Lladser

YOUR NAME: ANSWER KEY

(a) Assuming that Colorado has a population of 5 millions, respond: what is the least number of people in Colorado with the same last two SSN-digits and born on the same day of the year (but not necessarily in the same year)? To respond assume that any pair of digits may occur as the last two SSN-digits, and recall that a leap year has 366 days. **You do not need to simplify your answer as long as it clearly specifies a nonnegative integer.**

$$\left\lceil \frac{5\times 10^6}{100\cdot 366} \right\rceil = \lceil 136.6\cdots \rceil = 137$$ i.e. 137 coloradians share the same b-day and their last two SSN-digits.

(b) What's the general solution to the recursion: $a_n = 4a_{n-1} - 3a_{n-2}$, with $n \geq 2$?

The characteristic eq. assoc. with this linear homogeneous recursion with constant coefficients is

$r^2 = 4r - 3 \iff r^2 - 4r + 3 = 0 \iff r = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2}$

i.e. $r = 1$ or $r = 3$. Hence

$$a_n = \alpha + \beta \cdot 3^n, \quad n \geq 0$$

(c) What's the general solution to the recursion: $a_n = 4a_{n-1} - 4a_{n-2}$, with $n \geq 2$?

This is also a LHR w/ CC's. The associated characteristic equation is

$r^2 = 4r - 4 \iff r^2 - 4r + 4 = 0 \iff r = \frac{4 \pm \sqrt{16-16}}{2} = 2$

which has multiplicity 2. Hence

$$a_n = (\alpha + \beta \cdot n) \cdot 2^n, \quad n \geq 0$$