APPM 3170: Discrete Applied Mathematics - Fall 2008

Quiz #5
Lecturer: Manuel Lladser

YOUR NAME: ANSWER KEY

(b) Determine a closed formula for the sequence determined by the generating function

$$\frac{x^3}{1-5x}.$$

$$\frac{x^3}{1-5x} = x^3 \sum_{k=0}^{\infty} (5x)^k = \sum_{k=0}^{\infty} 5^k x^{k+3} = \sum_{k=3}^{\infty} 5^{k-3} x^k$$

Hence $$\boxed{[x^k]\ \frac{x^3}{1-5x} = \begin{cases} 0, & k<3 \\ 5^{k-3}, & k\geq 3 \end{cases}}$$

(a) Consider two sets A and B. How many elements does $(A \cap B)$ have if $|A| = 15$, $|B| = 17$, and $|A \cup B| = 19$?

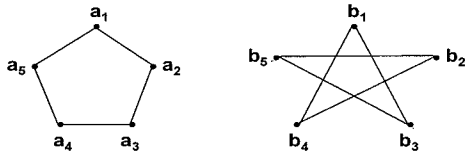
According to the inclusion-exclusion principle:

$$|A \cup B| = |A| + |B| - |A \cap B| \quad \text{i.e.}$$

$$|A \cap B| = |A| + |B| - |A \cup B| = 15 + 17 - 19 = \boxed{13}$$

(c) Are the following graphs are isomorphic? Explain.

Yes! Just define $f(a_1) = b_1$, $f(a_2) = b_3$, $f(a_5) = b_4$, $f(a_3) = b_5$ and $f(a_4) = b_2$. f is clearly a bijection. Furthermore, it preserves edges as the following tables show



Edges on LHS	f(Edges on LHS)	Edges on RHS?
$\{a_1, a_2\}$	$\{b_1, b_3\}$	✓
$\{a_2, a_3\}$	$\{b_3, b_5\}$	✓
$\{a_3, a_4\}$	$\{b_5, b_2\}$	✓
$\{a_4, a_5\}$	$\{b_2, b_4\}$	✓
$\{a_5, a_1\}$	$\{b_4, b_1\}$	✓