APPM 3310 - Review for Exam 2

1. Vector spaces

   - Be able to give the definition of a vector space, as described in Strang (p. 64 boldprint and p. 69, problem 2.1.5, box).
   - Be able to give examples of several different types of vector spaces, including function spaces.
   - Be able to define a vector subspace. Be able to verify that a given subset of a vector space is a subspace. Be able to give examples of nontrivial subspaces of vector spaces.
   - Be able to verify that a collection of vectors in $\mathbb{R}^n$ is independent/dependent.
   - Know the definition of a basis of a vector space. Given a collection of vectors in $\mathbb{R}^n$, be able to determine whether or not the set is a basis for $\mathbb{R}^n$.
   - Know the four fundamental subspaces associated with any $m \times n$ matrix. Be able to find a basis for each subspace.
   - Be able to state precisely the fundamental theorem of linear algebra, as described in Strang (p. 95 and p. 138 - box 3D).
   - Know the rank-nullity equation and understand how it relates to the fundamental theorem of linear algebra.
   - Be able to describe precisely the relationship between the four fundamental subspaces and existence and uniqueness of solutions to $Ax = b$, as explained in Strang (p. 96 - box 2Q).
   - Know the relationship between the dimensions of the vector subspaces U, V, $U + V$, and $U \cap V$.

2. Linear transformations

   - Be able to provide examples of projection, reflection, rotation, and dilation matrices.
   - Be able to state the basic properties of these transformations, in terms of their inverses and powers.

- Understand how to express a linear transformation by it's action on basis vectors.

3. Geometry of $\mathbb{R}^n$

   - Understand how to use the dot product to compute lengths of vectors and angles between vectors.
   - Know the basic properties of the dot product.
   - Know how to project a given vector onto another vector using the dot product. Using this projection, know how to compute a decomposition of the original vector into orthogonal components.
   - Know how to compute the matrix that projects any vector onto a given vector.

4. Orthogonality

   - Know what it means for two vector subspaces to be orthogonal and be able to give a nontrivial example.
   - Know what it means for a subspace to be the orthogonal complement of another subspace and be able to give a nontrivial example.

5. Least Squares

   - Know the normal equations and when they can be solved to compute the least squares solution to $Ax = b$.
   - Understand how the normal equations give the matrix which projects any vector onto the column space of A.
   - Be able to compute the least squares solution of a simple $Ax = b$ problem.

6. Review problems

   Chapter 2 review problems : 1,3,8,9,12,14,15,16,19,22,28,29
   Chapter 3 review problems : 1,3,4,8,29,34