APPM 3310 - new material for the final

1. Gram-Schmidt and $A = QR$ factorization
   - Be able to perform the Gram-Schmidt procedure to transform a collection of vectors into an orthonormal set of vectors with the same span.
   - Know how to write the original vectors in terms of the new orthonormal vectors.
   - If the original collection of vectors are columns of a matrix A, know how to deduce the $A = QR$ factorization from this information.
   - Know that if A has independent columns, then A can be factored QR.
   - Know what an orthonormal matrix is. Know what an orthogonal matrix is. Be able to provide examples of each.
   - Know what it means for orthonormal matrices to preserve length.
   - Understand how the normal equations for the least squares solution to $Ax = b$ simplify to $Rx = Q^T b$ if A is factored $A = QR$.
   - Understand why this formulation could be advantageous for numerical computations.
2. Eigenvalues, eigenvectors, and diagonalization
   - Know how to compute the eigenvalues and eigenvectors of a square matrix.
   - Know that the sum of eigenvalues is the trace of the matrix and the product of eigenvalues is the determinant.
   - Know what the characteristic polynomial of a matrix is.
   - Know the statement of the Cayley-Hamilton theorem.
   - Know the definitions of algebraic and geometric multiplicity of an eigenvalue.
   - Know that the algebraic multiplicity is always greater than or equal to the geometric multiplicity.

- Be able to give examples of eigenvalues with equal algebraic and geometric multiplicity, and algebraic multiplicity strictly great than geometric multiplicity.
- Know that eigenvectors corresponding to different eigenvalues are independent.
- Know that if the algebraic multiplicity equals the geometric multiplicity for each eigenvalue, then the matrix is diagonalizable. In this case, know that the matrix of eigenvectors S satisfies $A = SDS^{-1}$.
- Understand how the forumla for A^k simplifies upon diagonalization of A.

3. Linear difference equations

   - Be able to write down an expression for the general solution of $u_{k+1} = Au_k$, where A is a square matrix. Know how the expression simplifies if A is diagonalizable.
   - Be able to compute $\lim_{k\to\infty} u_k$, the steady state behavior of the system, using the above expression.
   - Understand why the only equilibrium solutions correspond to eigenvectors for the eigenvalue 1.

4. Linear differential equations

   - Be able to write down an expression for the general solution of $u' = Au$, where A is a square matrix.
   - Be able to write down a formula for the matrix exponential $exp(At)$ when A is diagonalizable.
   - Understand why $u_0 = 0$ is the only equilibrium solution for the system. Understand how it's stability is determined by the real parts of the eigenvalues of A.