APPM 3310: Matrix Methods — Exam #1 — Sept 30, 2009

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. Show all work in your bluebook. A correct answer with no supporting work may receive **no credit** while an incorrect answer with some correct work may receive partial credit. One page of notes is permitted, but no other books or electronic devices are allowed.

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1. (30 points) True/false questions. In each case, you must also give a short **explanation** of your answer.

   (a) If A and B are square matrices and $AB = I$ then $BA = I$.

   (b) If $A = LU$ gives the LU-decomposition of A, then $\det(A) = \det(L)$.

   (c) If A and B are nonsingular $n \times n$ matrices, $(A + B)^{-1} = A^{-1} + B^{-1}$.

   (d) If A is a nonsingular $n \times n$ matrix, $(A^T)^{-1} = (A^{-1})^T$.

   (e) If A is a singular $n \times n$ matrix then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

2. (40 points) Consider the system of equations with $A\mathbf{x} = \mathbf{b}$ with $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & -2 & 7 & 6 \\ -1 & -2 & 2 & 1 \end{pmatrix}$ and $\mathbf{b}$ an arbitrary vector.

   (a) Use Gaussian elimination to transform the augmented matrix $(A|\mathbf{b})$ into row echelon form.

   (b) Find the LU decomposition of A.

   (c) What is $\operatorname{rank}(A)$?

   (d) Define "range" and find a basis for the range of A.

   (e) Define "kernel" and find a basis for the kernel of A.

   (f) Set $\mathbf{b} = (0, 2k, 3)^T$. For what values of k does the system $A\mathbf{x} = \mathbf{b}$ have a unique solution? no solution? infinitely many solutions?

3. (30 points)

   (a) Give the definition of "basis" of a vector space.

   (b) Explain why the set of all continuous functions such that $f(1) = 0$ forms a (vector) subspace of the continuous functions, but the set of functions such that $f(0) = 1$ does not.

   (c) Let $R \subset \mathcal{M}_{m\times n}$ consist of those matrices that have rank 2. Is R a vector subspace of $\mathcal{M}_{m\times n}$? Why or why not.

   (d) Are the vectors 1, $2x - 1$, $(x - 1)^2$, $x^2 \in P^{(2)}$ independent? Why or why not?

   (e) Suppose $S = \operatorname{span}(1, 2x - 1, (x - 1)^2, x^2)$. What is the dimension of S?

   (f) Find a basis for S.