3310 Fall 2009 Exam 1 Section 1

① 4 pts T/F + 2 pts explanation

a) T $\quad B = A^{-1}$

b) F $\quad \det(A) = \det(U)$

c) F $\quad (1)^{-1} + (2)^{-1} \neq (1+2)^{-1}$ even 1×1 case!

d) T $\quad$ Transpose & inverse commute $\quad AA^{-1} = I \Rightarrow (A^{-1})^T A^T = I \Rightarrow (A^{-1})^T = (A^T)^{-1}$

e) F $\quad$ System might be inconsistent

② 7 ⓐ
$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & b_1 \\ -1 & -2 & 7 & 6 & b_2 \\ -1 & -2 & 2 & 1 & b_3 \end{array}\right) \underset{R_1+R_3 \to}{\xrightarrow{R_1+R_2 \to}} \xrightarrow{-\frac{1}{2}R_2+R_3 \to} \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & b_1 \\ 0 & 0 & 10 & 10 & b_2+b_1 \\ 0 & 0 & 0 & 0 & b_3 - \frac{1}{2}b_2 + \frac{1}{2}b_1 \end{array}\right)$$

7 ⓑ
$$LU = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & \frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 \end{pmatrix} = A$$

5 pts ⓒ $\operatorname{rank}(A) = 2$ (2 pivots)

7 ⓓ $\operatorname{rng}(A) = \{b : Ax = b \text{ for some } x \in \mathbb{R}^n\}$

two ways: pivot columns $\operatorname{rng} A = \operatorname{span}\underbrace{\left(\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix}\right)}_{\text{basis}}$

or solve consistency $b_1 = b_2 - 2b_3 \Rightarrow \operatorname{rng}(A) = \operatorname{span}\overbrace{\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}\right)}^{\text{basis}}$

7 ⓔ $\ker(A) = \{z : Az = 0\}$

$z_3 = -z_4$

$z_1 = -2z_2 - 3z_3 - 4z_4 = -2z_2 - z_4$

$$\ker(A) = \operatorname{span}\left(\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}\right)$$

7 ⓕ consistency says $0 = 2k - 2\cdot 3 \Rightarrow k = 3$

unique solution $\Rightarrow$ never

no solution $\Rightarrow k \neq 3$

only many solution $\Rightarrow k = 3$

\#3 a) A basis of V is a set of independent vectors that span V.

b) The set of C^0 functions s.t. $f(1) = 0$ is closed under addition and scalar multiplication (since $c_1 f_1(1) + c_2 f_2(1) = 0$ if $f_1(1) = f_2(1) = 0$)
but the set for which $f(1) = 1$ is not ($c_1 f_1(1) + c_2 f_2(1) = c_1 + c_2 \neq 1$)

c) Matrices of rank(2) are not a subspace since, for example they don't contain the zero matrix.

Also not closed under addition (e.g. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$)
or scalar multiplication ($cA = 0$ if $c = 0$)

d) We are given 4 vectors in $P^{(2)}$ but $\dim(P^{(2)}) = 3$, so they cannot be independent.

e) Set $p = c_1(1) + c_2(2x-1) + c_3(x-1)^2 + c_4 x^2 = a_0 + a_1 x + a_2 x^2 \in P^{(2)}$
we can show that any vector in $P^{(2)}$ can be reached. One way is to see that the matrix of coefficients

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & 0 & a_0 \\ 0 & 2 & -2 & 0 & a_1 \\ 0 & 0 & 1 & 1 & a_2 \end{array}\right)$$

has rank $= 3 \Rightarrow \dim(S) = 3$

f) since $S = P^{(2)}$ we can choose $(1, x, x^2)$ as a basis or, if you like the pivot columns of the coefficient matrix correspond to the vectors $(1, 2x-1, (x-1)^2)$