APPM 3310: Matrix Methods
Section 01 — Exam #2 — Nov. 11, 2009

On the front of your bluebook print (1) your name, (2) your section number, (3) your instructor name, and (4) a grading table. Show all work in your bluebook. Textbooks, class notes and calculators are not permitted. If you find that the arithmetic for this exam seems complicated, go back and check your work.

Please sign your bluebook under the Honor Code to indicate that you have neither given nor received unauthorized assistance on this exam, and that you will not talk with ANYONE about this exam until after 3PM on 11/11/2009.

1. (40 points) For this problem, assume A is a 4×4 matrix with:

$$\text{rng}(A) = \text{ span } \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right\}.$$

   (a) Find an orthonormal basis for $\text{rng}(A)$.

   (b) Find rank(A), $\dim(\ker A)$, $\dim(\text{coker } A)$, $\dim(\text{rng } A)$, and $\dim(\text{corng } A)$.

   (c) Find a basis for $\text{coker } A$.

   (d) What are the solvability (compatibility) condition(s) for the linear system $A\mathbf{x} = \mathbf{b}$? Explain.

2. (40 points)

   (a) Is the Gram matrix of $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ positive definite? Why or why not?

   (b) Let $\mathbf{v}_1, \cdots, \mathbf{v}_4$ be vectors in $\mathbb{R}^3$. Can their Gram matrix be positive definite? Why or why not?

   (c) Is the matrix $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 2 & 5 \end{pmatrix}$ positive definite? (Don't forget: show your work!)

   (d) Is the expression $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + u_1 v_2 + u_2 v_2$ an inner product for vectors in $\mathbb{R}^2$? Why or why not?

3. (20 points) Find the least squares solution to the linear system:

$$\begin{pmatrix} 3 & -1 \\ 0 & 2 \\ -2 & 1 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$