APPM 3310: Matrix Methods — Exam #2 — Nov. 11, 2009

On the front of your bluebook print (1) your name, (2) your section number, (3) your instructor name, and (5) a grading table. Show all work in your bluebook. Textbooks, class notes and calculators are not permitted. If you find that the arithmetic for this exam seems complicated, go back and check your work.

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1. (40 points) For this problem, assume A is a 4×4 matrix with:

$$\text{rng}(A) = \text{ span } \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right\}.$$

   (a) Find an orthonormal basis for rng(A).

   (b) Find rankA, dimkerA, dimcokerA, dimrngA, and dimcorngA.

   (c) Find a basis for cokerA.

   (d) Would $Ax = b$ have a solution if $b = (1, 1, 2, 0)^T$? What if $b = (0, 2, 2, 0)^T$? Explain.

2. (40 points)

   (a) Is the Gram matrix of $v_1 = (1, 1)^T$ and $v_2 = (1, -1)^T$ in $\mathbb{R}^2$ positive definite? Why?

   (b) Let $v_1, \cdots, v_6$ be vectors in $\mathbb{R}^4$, can their Gram matrix be positive definite? Why?

   (c) Is the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 9 \end{pmatrix}$ positive definite?

   (d) Is the expression $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + 2u_1 v_2 + 3u_2 v_2$ an inner product for vectors in $\mathbb{R}^2$? Why or why not?

3. (20 points) Find the closest point to $b = (1, 2, -1, 3)^T$ in the subspace $W = span\{(1, 0, 2, 1)^T, (1, 1, 0, -1)^T\}$.