

## Student Project, APPM 3310 — Fall 2009

Project due dates: *Oct. 30, Nov. 20, Dec. 7*

We have studied some of the foundations of linear algebra and have hinted at different types of applications. In this project, you will choose an application of matrix methods that is of interest to you and prepare a paper on this topic. Some possible topics (in no particular order) include:

1. Graphs and incidence matrices from section 2.6.
2. Image compression. This is a huge area of active research, but it is fundamentally a problem in linear algebra. If you are interested in this, you will need to restrict to a specific subtopic. A typical data compression scheme has 3 components: a transform, quantization, and arithmetic coding. The transforms that are used are linear algebraic transformations with special properties. You may want to understand their properties and their relationship to the other steps.
3. There are many “named” matrix families. These include: Hadamard, Hankel, Hilbert, Magic, Pascal, Rosser, Toeplitz, and Vandermonde. You can investigate one of these families—applying many of the techniques that we are learning—and discuss their application.
4. Tensors are a further generalization of matrices and are useful in physics (e.g. elasticity, general relativity, and many other fields). What is a tensor and how do they generalize matrices? What matrix operations can become tensor operations?
5. Linear algebra for structural mechanics, e.g. see sections 6.1, 6.3, or 9.5; or linear algebra for electrical networks, e.g. see section 6.2
6. Additional topics in chapters 9 and 11.
7. Linear programming. (This is a huge area, you’ll need to restrict to one small set of examples.)
8. Linear algebra in problems of optimization, including economic models. (This is a huge area, you’ll need to restrict to a specific type of algorithms or subset of problems.)
9. Quaternions form a number field, a further generalization of complex numbers. In spite of the efforts of Hamilton, Maxwell, and many others, so far there is no significant application of them to physics. However, they are widely used in graphics as an efficient way to perform rotations. You may want to investigate this.
10. Network models and spanning trees—such models are important for an understanding of the internet, of power distribution (and blackouts), etc.
11. Wavelets are based on matrix methods. You might investigate the Haar wavelet family. There are several “introductory” texts on wavelets, including *Discovering Wavelets* by Edward Aboufadel and Steven Schlicker.
12. The vector space model is fundamental to a basic understanding of search engines. *Understanding Search Engines* by Michael Berry and Murray Browne. is a place to start.

13. The fast Fourier transform (FFT) is a mainstay in modern technology, e.g. tomography, synthetic aperture radar, etc. This may be a good entry point to learn about fast algorithms. You can begin with section 5.7.

These topics are not exhaustive. Come talk with me about your ideas. I'd like you to choose a topic that interests you!

The paper will proceed in three steps. The first part will be a **2-page project proposal** (Due Oct. 30). The project proposal will include

- an introduction to your topic,
- some background information,
- an outline of what you will cover in your paper, and
- a start on your list of references.

The second part will be a **rough draft** (Due Nov. 20) and the third part will be the **final paper** (Due Dec. 7). Your paper should not be a random collection of facts. You should identify at least one specific question/problem, and the goal of your paper is to answer that question. Both the rough draft and the final project will be written in a manner similar to a research article in a scientific journal. Your paper must be typewritten, although lengthy derivations can be handwritten (neatly!!) and included in an appendix. Your paper should have the following sections, as they pertain to your problem.

**Abstract:** A short (1-2 paragraphs) summary of your problem and results—this is an advertisement to entice the reader to proceed further.

**Introduction/Summary:** This section serves to introduce your topic, provide any background information, and summarize your results. Include a survey of other people's work and/or a short discussion of the existing literature with references, as appropriate. Provide all necessary definitions and concepts for me (presumably ignorant of your particular application) to understand the question you are studying. The introduction should conclude with a short summary of your findings: what did you find? Is it what you had expected? How can you use your results? What do they tell you about your original question? (This section may be anywhere from 1-3 pages long.)

**Model development/mathematical formulation:** This section serves to develop your model. All symbols should be defined/identified. Any approximations you make in going from your physical model to the mathematical model should be noted. Lengthy derivations can be included here or placed in an appendix. Results (and proofs, where appropriate) can be included in this section or can stand alone in their own section.

**Numerical Work (if appropriate) or Examples:** This section describes your algorithms and/or numerical work and/or examples. A descriptive summary of your code and results can be included here. You can put any (well-documented!) code that you write in the appendix. Graphs and/or tables with data you used or computed should be in this section.

**References:** All books, articles, and websites used in the preparation of your paper should be documented in this section. Note that if you do not properly document your references and attribute the results that you used to the proper sources, then your work is plagiarism.

Additional items to keep in mind:

- You are strongly encouraged to work with a partner. If you absolutely can not find someone to work with, you may work alone.
- Cover pages are not needed, just a title, date, and your name at the top of the first page. You are encouraged to print two-sided.
- All facts that you state in your paper must have a reference. Include all references at the end of the paper. You may refer to references in the text by number (e.g. [1]), or by name and year (e.g. (Jones, 1986)).
- Give complete definitions of all terms and symbols used.
- Figures, graphs, and tables should have labels, captions and legends, as appropriate. You should discuss each figure in the text, as in “We see from Figure 3 that ...” If you do not discuss it in the text, it should not be included in the report!

This project is worth 100 points toward your final grade. The proposal, due on Oct. 30, will be worth 10 points, and the rough draft, due on Nov. 20, 20 points. Each of the following elements will be weighted somewhat equally:

- Writing style, clarity and completeness of explanations.
- Problem choice, mathematical development and explanation of the problem.
- Numerical development and/or examples, explanation and discussion. In general, code should be in an appendix and graphs and figures should be in the text where they are first discussed.