Exam I

APPM 3310

Spring 2002

1. Find a basis for the subspace of $\mathbb{R}^4$ that contains all vectors such that $x_1 = 2x_3$ and $x_2 = -x_4$. Show that your basis actually *is* a basis by checking the necessary properties.

2. Let $C = AB$, where A and B are symmetric, $n \times n$ matrices. Compute C^T. Is C symmetric, in general? Justify your answer.

3. Find the value of c that makes the following system solveable:

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & -1 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ c \end{bmatrix}.$$

   There is no need to compute the solution x.

4. Compute the inverse of the following matrix:

$$A = \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ \sqrt{3}/2 & 0 & -1/2 \end{bmatrix}.$$

   What is A^3?

5. Say whether the following statements are true or false. No justification is necessary.

   (a) If A has n columns and m rows then the nullspace of A is a subspace of $\mathbb{R}^m$.

   (b) Matrix multiplication cannot be carried out with two $m \times n$ matrices if $m \neq n$.

   (c) The zero vector is an element of every subspace of a vector space.

   (d) Let A be an $n \times n$ matrix. If $Ax_1 = Ax_2$, with $x_1 \neq x_2$, then A is not invertible.