Exam II
APPM 3310
Spring 2002

Show your work or explain your reasoning for each answer.

1. Suppose $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix} \right\}$ is a basis for a subspace V of $\mathbb{R}^4$, and let $V^\perp$ denote the orthogonal complement of V.

   (a) What is the dimension of $V^\perp$? (Show the calculation, explain the reasoning.)

   (b) Find a basis for $V^\perp$.

   (c) Construct the matrix of projection onto $V^\perp$.

2. A matrix P is a projection matrix if and only if it satisfies two properties.

   (a) What are the two properties?

   (b) If P is a projection matrix, show that $P_1 = I - P$ is also a projection matrix. (Hint: Show that P_1 satisfies the two properties you gave in answer to the previous question. If you do not know these properties, I will sell them to you for 5 points each.)

3. Suppose $\{\alpha_1, \alpha_2\}$ is a basis for a subspace W of $\mathbb{R}^4$, where

$$\alpha_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \qquad \alpha_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}.$$

   Construct an orthonormal basis $\{q_1, q_2\}$ for W.

4. For $x, y \in \mathbb{R}^n$, show that

$$\frac{1}{2}\left(\|x+y\|^2 + \|x-y\|^2\right) = \|x\|^2 + \|y\|^2 .$$