
Linear Algebra
Practice Midterm 2

1. Consider $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 0 & 1 & -3 \\ 2 & 1 & -7 \\ 1 & 0 & -2 \end{bmatrix}$.

- (a) Find the echelon form of A .
- (b) Find the dimension of the four fundamental subspaces of A .
- (c) Find a basis for each of the four fundamental subspaces of A .
- (d) Use your bases to show explicitly that the row space of A is orthogonal to the null space of A .

2. Consider $B = \begin{bmatrix} 1 & -1 \\ 2 & -2 \\ 0 & 1 \end{bmatrix}$.

- (a) Use Gram-Schmidt on the columns of B to produce an orthonormal set.
- (b) Find the QR factorization of B .
- (c) The column space of B is a subspace of what space?
- (d) Find the matrix which projects onto the column space of B .

(e) Find the projection of the vector $\mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto the column space of B .

- (f) Is the system $B\mathbf{x} = \mathbf{a}$ consistent? Why or why not?

(g) Find the projection of the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ onto the column space of B .

- (h) Is the system $B\mathbf{x} = \mathbf{b}$ consistent? Why or why not?

3. A linear transformation in \mathbf{R}^3 stretches the x -axis by a factor of 3, shrinks the y -axis by a factor of 2, and projects onto the xy plane.

- (a) Find the matrix of this linear transformation.
- (b) Describe the column space and null space of this transformation.
- (c) Is this transformation invertible?

4. (a) What is the rank of the $n \times n$ matrix with every entry equal to 1?

(b) What is the rank of the $n \times n$ checkerboard matrix with $a_{ij} = 0$ for $i + j$ even and $a_{ij} = 1$ for $i + j$ odd?

5. Miscellaneous Proofs.

(a) If A is an $n \times n$ matrix of rank n and $A^2 = A$, prove that $A = I$.

(b) If you have a set of vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ which are an orthonormal basis for \mathbf{R}^n , prove that $\mathbf{v}_1\mathbf{v}_1^T + \mathbf{v}_2\mathbf{v}_2^T + \dots + \mathbf{v}_n\mathbf{v}_n^T = I$.

(c) Prove that if a set of vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ are orthonormal, they must be linearly independent.