

APPM 3310: Matrix Methods — Exam #1 — September 28, 2005

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. Show all work in your bluebook. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes and calculators are not permitted.

Please sign your bluebook under the Honor Code to indicate that you have neither given nor received unauthorized assistance on this exam.

1. (50 points) For this problem, use the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & -1 & -1 \\ 3 & 2 & 5 \end{pmatrix}$

- (a) Find the LU factorization of A , $A = LU$.
- (b) Identify the pivot elements of U .
- (c) What is $\text{rank}(A)$?
- (d) Find the general solution to the system $A\mathbf{x} = \mathbf{0}$.
- (e) For what values of k does the system $A\mathbf{x} = \mathbf{b}$ for $\mathbf{b} = (1, 2, -1, k)^T$, have a solution? Find the general solution(s) for these values of k .
- (f) Give the definition for the kernel of an $m \times n$ matrix. Find a basis for $\ker A$.
- (g) Give the definition for the range of an $m \times n$ matrix. Find a basis for $\text{rng}(A)$.
- (h) What is $\dim(\text{coker}(A))$?
- (i) What is $\dim(\text{corange}(A))$?

2. (50 points) A few unrelated, short answer questions.

- (a) Give the definition for W to be a subspace of a vector space V . Is the set of $n \times n$ matrices with $\det A = 0$ a subspace of the vector space $M_{n \times n}$? Explain.
- (b) Are the polynomials $p_1 = x^2 + 1$, $p_2 = (x - 1)^2$ linearly independent? Do they span $\mathcal{P}^{(2)}$? Explain.
- (c) For which value(s) of k does the system

$$\begin{aligned}x + ky &= 4 \\ kx + y &= 4\end{aligned}$$

have (i) no solution, (ii) exactly one solution, or (iii) infinitely many solutions?

- (d) If A and B are square matrices and $AB = I$, does $BA = I$? (Show this is true or provide a counterexample.)
- (e) Show that if C is any $m \times n$ matrix, then $C^T C$ is a symmetric matrix. (A complete answer will include the definition of a symmetric matrix.)