APPM 3310: Matrix Methods — Exam #2 — November 11, 2005

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. Show all work in your bluebook. Textbooks, class notes and calculators are not permitted, although you are allowed to use an index card reminder sheet. If you find that the arithmetic for this exam seems complicated, go back and check your work.

Please sign your bluebook under the Honor Code to indicate that you have neither given nor received unauthorized assistance on this exam.

1. (40 points) For this problem, assume A is a 4×3 matrix.
   (a) State the complete Fundamental Theorem of Linear Algebra.
   (b) Suppose you know that $\text{corng}(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \right\}$, find a basis for $\ker(A)$.
   (c) Find an orthonormal basis for $\text{corng}(A)$.
   (d) Use your results from part (c) to find the QR decomposition for $B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix}$.

2. (40 points) Each of the following unrelated questions involve some concept from class. The concept is in bold-face. A complete answer for each part will include a definition of the bold-faced word.
   (a) Is the expression $\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 - 3u_1v_2 - 2u_2v_1 + v_1v_2$ an **inner product** for vectors in $\mathbb{R}^2$? Why or why not.
   (b) If $P = P^TP$, show that P is a **projection matrix**.
   (c) Verify the **Cauchy-Schwarz inequality** for the functions $f(x) = x^3$, and $g(x) = x$ using the L^2 inner product on the interval $(-1, 1)$.
   (d) Suppose A is an $m \times n$ matrix with $\text{rank}(A) = n$. Prove that $P = A^TA$ is a **positive definite matrix**.
   (e) Suppose C is a positive definite, **orthogonal** matrix. Find $\det(C)$.

3. (20 points) The following data corresponds to the depth d in inches snowfall in Denver for the winters starting in years $2000 + t$

t	0	1	2	3	4
d	58.3	30.2	61.8	38.0	39.3

   Your task is to decide whether this data provides evidence for global warming or not.
   (a) Set up (DO NOT SOLVE) the least squares problem to show that the snowfall has been approximately linearly decreasing recently (You may use d_i and t_i to represent the depths and times). Your solution will involve writing down a linear system whose solution will give the least squares fit.
   (b) What quantity would you use to estimate the error in your approximation?