APPM 3310: Matrix Methods — Exam #1 — February 27, 2006

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. Show all work in your bluebook. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes and calculators are not permitted.

Please sign your bluebook under the Honor Code to indicate that you have neither given nor received unauthorized assistance on this exam.

1. (50 points). Each answer for the following statements is True or False. Your answer **must** be justified. If the answer is True then you must explain why it is True; if it is False you must give a counterexample. Assume all matrices in this problem are square.

   (a) For two matrices A and B, if $AB = \mathbf{0}$ then $A = \mathbf{0}$ or $B = \mathbf{0}$.

   (b) If A is a nonsingular matrix then $AB = \mathbf{0}$ implies that $B = \mathbf{0}$.

   (c) If C and D are symmetric matrices, so is CD.

   (d) If $A = LU$, then $\det(A) = \det(U)$.

   (e) If A is a 3×3 matrix and the equation $A\mathbf{x} = (1, 0, 0)^T$ has a unique solution, then A is invertible.

2. (20 points) Let V be the set of polynomials in $P^{(3)}$ that satisfy $p(1) = 0$ and $p(-1) = 0$. That is, $V = \{p(x) = ax^3 + bx^2 + cx + d \mid p(-1) = p(1) = 0\}$

   (a) Show that V is a subspace of $P^{(3)}$. (A complete answer for this question will include a definition of **subspace**.)

   (b) Find a **basis** for V.

3. (40 points) For this problem, use the matrix $A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 2 & 2 & -1 & 2 \\ 3 & 2 & -1 & 5 \end{pmatrix}$

   (a) Give the definition for the kernel of an arbitrary $m \times n$ matrix. Find a basis for $\ker A$ for A given in this problem.

   (b) Give the definition for the range of an arbitrary $m \times n$ matrix. Find a basis for $\operatorname{rng}(A)$.

   (c) State the Fundamental Theorem of Linear Algebra. Give the dimensions of each of the four fundamental subspaces of matrix A.

   (d) For what values of k does the system $A\mathbf{x} = \mathbf{b}$ for $\mathbf{b} = (1, k, 2)^T$, have a solution? Find the general solution(s) for these values of k.