

APPM 3310: Matrix Methods — Exam #2 — April 17, 2006

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. Show all work in your bluebook. Textbooks, class notes and calculators are not permitted. If you find that the arithmetic for this exam seems complicated, go back and check your work.

Please sign your bluebook under the Honor Code to indicate that you have neither given nor received unauthorized assistance on this exam.

1. (40 points) For this problem, assume A is a 3×4 matrix, except for part (a).
 - (a) State the complete (parts 1 and 2) Fundamental Theorem of Linear Algebra for a general $m \times n$ matrix A .
 - (b) Suppose you know that $\text{rng}(A) = \text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \right\}$, find a basis for $\text{coker}(A)$.
 - (c) Find an orthonormal basis for $\text{rng}(A)$.
 - (d) Would $Ax = b$ have a solution if $b = (1, 1, 1)^T$? What if $b = (-4, 1, 1)^T$? Explain.
 - (e) Use your results from part (c) to find the QR decomposition for $B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix}$.
2. (40 points) Each of the following unrelated questions involve some concept from class. The concept is in bold-face. A complete answer for each part will include a definition of the bold-faced word.
 - (a) Is the expression $\langle \mathbf{u}, \mathbf{v} \rangle = u_1v_1 + u_1v_2 + u_2v_1 + v_1v_2$ an **inner product** for vectors in \mathbb{R}^2 ? Why or why not?
 - (b) What are the L^1 and L^∞ **norms** on $[0, 1]$ for the function $f(x) = (1/3) - x$?
 - (c) Write the quadratic form $q(x, y, z) = x^2 + 4xy + 6y^2 - 2xz + 9z^2$ in the form $q(\mathbf{x}) = \mathbf{x}^T K \mathbf{x}$ for some symmetric matrix K . Is K **positive definite**? What does this tell you about the extrema of $q(x, y, z)$?
 - (d) Show that if Q is an **orthogonal matrix**, then $\|Qx\| = \|x\|$ for any vector $x \in \mathbb{R}^n$, where $\|\cdot\|$ denotes the standard Euclidean norm.
3. (20 points) Consider the set of data given by: $y = 1$ at $t = -2$, $y = 0$ at $t = -1$, $y = 3$ at $t = 3$, and $y = 4$ at $t = 0$.
 - (a) Use the method of least squares to find the best straight line $y = \alpha + \beta t$ for this data.
 - (b) Compute the error of this approximation.