APPM 3310: Matrix Methods — Exam #1 — October 3, 2007

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. Show all work in your bluebook. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. No electronic devices of any kind (e.g. cell phones, calculators, etc.) are permitted.

Please sign your bluebook under the Honor Code to indicate that you have neither given nor received unauthorized assistance on this exam.

1. (10 points) Compute the determinant of matrix B in terms of λ, b_1, b_2, b_3 and b_4.

$$B = \begin{pmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & \lambda & -1 \\ b_1 & b_2 & b_3 & b_4 + \lambda \end{pmatrix}$$

2. (40 points) Let A be an $m \times n$ matrix.

   (a) Give the definitions for the range of A and the corange of A.

   (b) State the Fundamental Theorem of Linear Algebra.

   (c) Now find a basis and the dimension for the range, corange, and kernel of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

   (d) Use the matrix A in part (c) to give conditions on $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $A\mathbf{x} = \mathbf{b}$ has a solution.

   (e) Restate your answer in (d) using a subspace you found in part (c). (This should be no more than one or two sentences.)

3. (30 points) Let $\mathcal{P}^{(2)}$ be the set of polynomials of degree less than or equal to 2.

   (a) Show that $\mathcal{P}^{(2)}$ is a subspace of the vector space of all polynomials.

   (b) Give the definition of linear independence. Are the polynomials $p_1 = 1$, $p_2 = x$, and $p_3 = (-1 + 3x^2)/2$ linearly independent?

   (c) Give the definition of a basis. Are the polynomials in part (b) a basis for $\mathcal{P}^{(2)}$? Explain briefly.

4. (30 points)

   (a) Let $\mathbf{u} = (2, 1, 1)^T$ and $\mathbf{v} = (1, 0, 1)^T$. Compute $A = I - \mathbf{u}\mathbf{v}^T$ and find A^{-1}.

   (b) Now, let $\mathbf{u}$ and $\mathbf{v}$ be arbitrary $n \times 1$ vectors with $\mathbf{v}^T\mathbf{u} \neq 1$. Let $A = I - \mathbf{u}\mathbf{v}^T$. Give the definition for the inverse of a matrix and use it to verify that $A^{-1} = I + \dfrac{1}{1 - \mathbf{v}^T\mathbf{u}}\mathbf{u}\mathbf{v}^T$. (Hint: Remember that $\mathbf{v}^T\mathbf{u}$ is a scalar and $\mathbf{u}\mathbf{v}^T$ is a matrix.)