APPM 3310 Test 2 FALL 08 solns

1(a) See definition on p. 176 of Olver & Shakiban.

(b) Yes, you check.

(c) See definition p. 144

2. (a) See Theorem 3.5, p. 138.

(b) See Theorem 3.9, p. 142

(c) $\langle f, g \rangle = \int_{-1}^{1} xe^x \, dx = 2/e$ by integration by parts

& $\|f\| = \left(\int_{-1}^{1} x^2 dx \right)^{1/2} = \left(\frac{2}{3} \right)^{1/2}$, $\|g\| = \left(\int_{-1}^{1} e^{2x} \, dx \right)^{1/2} = \left(\frac{e^4 - 1}{2e^2} \right)^{1/2}$

Show $\frac{2}{e} \leq \sqrt{\frac{2}{3}} \sqrt{\frac{e^4-1}{2e^2}} = \sqrt{\frac{2(e^4-1)}{6e^2}} = \frac{\sqrt{2(e^4-1)}}{\sqrt{6}\, e}$.

Note $e \geq 2 \Rightarrow e^4 \geq 13 \Rightarrow 2(e^4-1) \geq 24 \Rightarrow \sqrt{2(e^4-1)} \geq 2\sqrt{6}$

$\Rightarrow \frac{\sqrt{2(e^4-1)}}{\sqrt{6}} \geq 2 \Rightarrow \frac{\sqrt{2(e^4-1)}}{\sqrt{6} \cdot e} \geq \frac{2}{e}$ ∎

3. (a) See definition p. 158

(b) See Theorem 3.28, p. 158.

(c) HW problem #3.4.29, see soln's to HW #7.

4. (a) Note, $K > 0 \Rightarrow K$ has positive pivots (see Thm 3.37, p. 164), and $\det(K)$ is equal to product of pivots so $\det(K) > 0$.

(b) Let $K = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$, suppose $a > 0$, suppose $ac - b^2 > 0$.

Note $ac - b^2 > 0 \Rightarrow ac \geq 0 \Rightarrow$ both a, c are non-negative or both are non-positive. Both a, c non-positive implies $a + c \leq 0$, a contradiction, so it must be that $a, c \geq 0$. Note if $a = 0$ then $\det(K) = -b^2 \leq 0$, a contradiction. So $a > 0$, and $ac - b^2 > 0$ so K is pos-def by Example 3.23, p. 155.

(c) False, not closed under scalar multiplication.

4 (d) HW problem #5.2.15, see HW 10 soln's.

5. Similar to HW problem # 5.2.7 (b), see HW 10 solns.

6. Similar to HW problem # 4.3.1, see HW 9 solns.