APPM3310 Matrix Methods

Semester Exam 1 **21 June 2006** **Total 100**

INSTRUCTIONS: You are allowed one, double sided A4 page with formulas. Calculators are not permitted. Work all problems and give the details of your calculations. A correct answer with the wrong argument may not receive any credit whereas a wrong answer with the correct argument might receive partial credit.

There are 5 questions, each question is worth 20 marks.

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1. Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix}.$$

   (a) Calculate the LU decomposition of A.

   (b) Use the LU decomposition of 1(a) and solve the system $A\mathbf{x} = \mathbf{b}$ using forward and back substitution, where

$$\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

2. Calculate the LU decomposition of

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 0 \end{bmatrix},$$

   i.e. calculate $PA = LU$ where P is a permutation matrix, L is an 3×3 lower triangular matrix and U is an 3×4 upper triangular matrix. Write down the elementary matrices as well as P, L and U.

3. Find *all* solutions of a system $A\mathbf{x} = \mathbf{b}$ where

$$\mathbf{b} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}.$$

   and $PA = LU$ where

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad \text{and } U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -1 \\ 0 & 0 & 0 & -2 \end{bmatrix}.$$

   Solve the system by using forward and back substitution. PTO

4. Find a basis for the span of the following vectors:

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v_3} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v_4} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

5. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

Find bases for all four fundamental subspaces of A.