APPM 3310: Matrix Methods — Exam #1 — October 1, 2008

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. **Explain all of your answers.** A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. No electronic devices of any kind (e.g. cell phones, calculators, etc.) are permitted. Begin each problem on a new page.

1. (30 points) For this problem, let $A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & -1 & 3 \\ 2 & 3 & 7 & 0 \end{bmatrix}$

   (a) Find the LU decomposition of A where U is in row echelon form.

   (b) Determine the rank of A and the dimensions of the four fundamental subspaces associated with A.

   (c) Find a basis for $\ker(A)$ and $\operatorname{rng}(A)$.

   (d) Let $\mathbf{b} = [1\ 0\ 3]^T$. Use the LU decomposition from the previous part to find a solution to the linear system $A\mathbf{x} = \mathbf{b}$

2. (10 points)

   (a) Describe three different ways you could tell whether a matrix is nonsingular.

   (b) Now, suppose you know A is $n \times n$ and $A\mathbf{u} = A\mathbf{v}$ for some $\mathbf{u} \neq \mathbf{v}$. Is A singular or nonsingular? **Explain.**

3. (20 points) Let $\mathcal{P}^{(3)}$ denote the vector space of all polynomials $p(x)$ of degree less than or equal to 3.

   (a) Are $p_1(x) = x^2 - 1$, $p_2(x) = x^2 + 1$, $p_3(x) = 5$, linearly independent elements of $\mathcal{P}^{(3)}$?

   (b) What is the dimension of the subspace of $\mathcal{P}^{(3)}$ spanned by p_1, p_2, p_3?

4. (10 points) A square matrix is *strictly upper triangular* if all of the entries on or below the main diagonal are zero.

   (a) Show that that the set of all strictly upper triangular matrices is a subspace of $M_{n\times n}$.

   (b) Is the same true for the set of all special upper triangular matrices? Justify your answer.

5. (20 points) Answer the following True/False questions by either showing why the statement is true in general or providing a counter example to show that it is false.

   (a) *True or False* A set of vectors is linearly dependent if the zero vector belongs to their span.

   (b) *True or False* The range of a matrix A is the same as the span of its columns.

   (c) *True or False* If A is an $m \times n$ matrix and $rngA = \mathbb{R}^m$ then $kerA = \mathbf{0}$.

6. (20 points) A matrix J is *skew-symmetric* if $J^T = -J$.

   (a) If J is skew-symmetric what do all of the entries on its diagonal have to be?

   (b) Write down an example of a skew-symmetric matrix.

   (c) Can you find a regular skew-symmetric matrix?

   (d) Prove that if J is a nonsingular skew-symmetric matrix then so is J^{-1}.